NBS Working paper
6/2019

Trend Inflation Meets Macro-Finance: The Puzzling Behavior of Price Dispersion

Aleš Maršál, Katrin Rabitsch, Lorant Kaszab

NATIONAL BANK OF SLOVAKIA
EUROSYSTEM

www.nbs.sk

research@nbs.sk

This publication is available on the NBS website
www.nbs.sk/en/publications-issued-by-the-nbs/research-publications

ISSN 2585-9269 (online)

Trend Inflation Meets Macro-Finance: The Puzzling Behavior of Price Dispersion *

Ales Marsal † Katrin Rabitsch‡ Lorant Kaszab §

July 29, 2019

Abstract

Motivated by recent empirical findings that emphasize low-frequency movements in inflation as a key determinant of term structure, we introduce trend inflation into the workhorse macro-finance model of Rudebusch and Swanson (2012). We show that this compromises the earlier model success and delivers implausible business cycle and bond price dynamics. We document that this result applies more generally to non-linearly solved models with Calvo pricing and trend inflation and is driven by the behavior of price dispersion, which is $i)$ counterfactually high and $ii)$ highly inaccurately approximated. We highlight the channels behind the undesired performance under trend inflation and propose several remedies.

Keywords: trend inflation; Calvo pricing; price dispersion; macro-finance; asset pricing;

JEL-Codes: E13, E31, E43, E44

---

*We wish to thank to our working paper referees Guido Ascari and Martin Andreasen for excellent comments. We have also benefited from discussions with Larry Christiano, and from feedback from Roberto Billi and Marcin Kolasa. The authors gratefully acknowledge financial support from the Austrian National Bank, Jubilaeumsfond Grant No. 17791.

†National Bank of Slovakia and Vienna University of Economics and Business. E-mail: ales.marsal@nbs.sk

‡Vienna University of Economics and Business. E-mail: katrin.rabitsch@wu.ac.at

§Magyar Nemzeti Bank and Vienna University of Economics and Business. E-mail: lorantkaszab1@gmail.com

1. Introduction

In recent years the macroeconomic profession has become increasingly aware of the importance of real-financial interactions and the need to explicitly incorporate a financial side into models used for economic policy analysis, such as the ones used at central banks. Of fundamental interest in this respect is the yield curve, which plays a crucial role for financial stability assessment. Although workhorse structural macro models have improved much in their ability to price financial assets (bond yields) these models still provide limited explanation of dynamics of prices and drivers of risk premia. A growing body of asset pricing literature highlights the importance of trend inflation in explaining U.S. bond price dynamics. For instance, Cieslak and Povala (2015) document that a large portion of movements in Treasury bond risk premia at business cycle frequencies can be attributed to low-frequency movements in inflation, i.e. trend inflation. Bauer and Rudebusch (2017) show that accounting for time-varying trend inflation (rather than variation in the cyclical component of inflation) stands as the key element in understanding the empirical dynamics of U.S. Treasury yields.[1]

Motivated by these crucial empirical findings and with the intent to improve our modeling frameworks that jointly address a macro and a finance side, we incorporate trend inflation into the macro-finance model of Rudebusch and Swanson (2012) (henceforth, RS). Their framework has evolved as a workhorse model, being the first to offer a resolution to a long-lasting struggle to explain why the term structure of interest rates is upward-sloping, the so called "bond premium puzzle" (cf. Backus, Gregory, and Zin (1989), and Den-Haan (1995)). Their New Keynesian model, enriched with Epstein-Zin preferences and long-run inflation risks, is successful at matching a large and variable term premium without compromising the model's ability to fit key macroeconomic variables. The mechanism that generates the sizable and time-varying term premium relies mostly on technology shocks which give rise to large inflation risks for bond holders at business cycle frequencies; a positive steady-state inflation rate plays no role as, in fact, the model is approximated around a zero-inflation steady state. We document that this assumption –abstracting from trend inflation– is not innocuous: instead of improving the model performance, the introduction of trend inflation compromises the earlier success of the RS model. The reason for this are drastically amplified inefficiencies from price rigidity as well as numerical inaccuracies that arise when the Calvo pricing mechanism meets trend inflation – in particular when using higher order approximations.

[1] Figure C, which reproduces Figure 1 of Rudebusch and Bauer (2017), summarizes these findings visually, by plotting time series for the ten-year yield, an estimate of trend inflation and the equilibrium nominal and real short rate.

Our paper offers an understanding of the channels behind these results and provides possible remedies.

The empirical literature, in both macro and finance, has long treated the inflation trend as constant. Stock and Watson (2007) provide strong evidence that the dynamics of inflation have been largely dominated by the trend component. Further, Cogley, Primiceri, and Sargent (2010) and Ascari and Sbordone (2014) demonstrate that inflation innovations account for a small fraction of the unconditional variance of inflation, implying that most of the volatility stems from the trend component of inflation. Similarly, also the theoretical literature accounts for trend inflation explicitly only relatively recently. Noteworthy examples are Ascari and Rossi (2012), Ascari, Castelnuovo, and Rossi (2011), Ascari and Ropele (2009), who show that trend inflation represents an important factor to consider in the design of monetary policy. As argued above, macro-finance models that are solved around a zero-inflation steady state may stand in contrast with the recent empirical evidence and it appears particularly important to realign current model frameworks with the empirical findings and incorporate positive trend inflation as a firm model element.[2]

We find that introducing trend inflation into the baseline RS model generates unrealistic business cycle and bond price dynamics. Moments from model simulated data become implausible, price dispersion and the implied output losses of price dispersion rise to unrealistic values, and price dispersion itself is inaccurately approximated. We document that the nonlinear behavior of price dispersion is at the core of the poor model performance when positive trend inflation meets Calvo pricing, and that problems are aggravated under decreasing returns to scale in the production function. It is important to emphasize that the encountered problems are not specific to the asset pricing related features of the RS model, but, more generally, apply to the class of macro models with a Calvo pricing mechanism and positive trend inflation, when solved non-linearly (under both second or third order approximations).[3] In fact, as we show throughout the paper, similar results can be obtained from a standard New Keynesian (NK) model of Clarida, Galí and Gertler (1999, hereafter CGG) under certain specifications and parameterizations, albeit generally to a lesser degree. The findings of this paper are, thus, relevant to more than just the macro-finance asset pricing literature and have broader

[2]It should be noted that also in the domain of macro-finance models some, few contributions have incorporated trend inflation, e.g., Andreasen, Fernandez-Villaverde, and Rubio-Ramirez (2018), Andreasen and Kronborg (2017), Kliem and Meyer-Gohde (2017) – typically, under the additional assumption of inflation indexation, which, as we will show, serves as one of our proposed remedies.

[3]However, while the problem is not specific to the macro-finance literature, the use of decreasing returns to scale production function (or, to be precise, a production function with fixed capital) in the context of a macro-finance model is no coincidence, as it contributes strongly to being able to match term premia; relaxing this assumption quickly leads to much smoother stochastic discount factors, and lower generated term premia.

applicability.[4,5]

The key contribution of our paper lies in the detailed formal explanation of the transmission and amplification mechanism between trend inflation, the distribution of prices, the real economy and bond prices. We emphasize two channels through which trend inflation has a key influence on model dynamics and drives up the model-implied levels of price dispersion: i) the marginal-cost channel, and ii) a trend-inflation-markup channel. Under the marginal-cost channel we understand the fact that firms that could not reset their prices recently and, because of trend inflation, are stuck with a (too) low price will employ an inefficiently large amount of labor and produce an inefficiently large amount of output; as these firms move along the concave production function to the right, their marginal cost will increase compared to aggregate marginal costs. Under the trend-inflation-markup channel we understand that firms incorporate trend inflation into their forward-looking pricing decision. They know that prices will go up and that they will not be able to change current prices for some period, so they set their optimal prices higher (at a markup in addition to the one from monopolistic competition) in the case of positive trend inflation compared to the case when trend inflation is zero. We demonstrate that the above channels lead to levels of price dispersion and implied output-losses from dispersion that lie significantly above values typically obtained in the case of zero trend inflation, and become counterfactually high. In addition, we show that the Calvo price dispersion equation also becomes poorly approximated by local perturbation methods.[6]

We propose several modeling devices that provide a remedy to the unrealistic business cycle and bond price moments, and to the inaccuracies to the price dispersion equation. In particular, an otherwise equivalent setup with Rotemberg price adjustment costs instead of Calvo pricing, a linear-in-labor production function, or the introduction of inflation indexation[7] can to a large degree restore the performance of the RS model (or more generally, a trend-inflation-augmented Calvo pricing model) in matching the

[4]E.g., the literature on globally solved ZLB-models under Rotemberg or Calvo, such as in Boneva, Braun, and Waki (2016) and Miao and Ngo (2019).

[5]There are arguably more realistic setups than the Calvo mechanism to capture nominal rigidities, such as discussed by the on state-dependent pricing (see, e.g., among others Golosov and Lucas (2007), Midrigan (2011), or Costain and Nakov (2011). Nonetheless, the Calvo mechanism, which belongs to the class of time-dependent pricing mechanisms, continues to remain the most widely used device to introduce nominal rigidities.

[6]We should note, that the problem of counterfactually price dispersion and its poor approximation, is present already in the original RS specification, without trend inflation. Positive steady state inflation, however, aggravates the problem substantially, up to the point that simulated model moments stop making sense. This issue of poor approximation has been raised also by Andreasen and Kronborg (2017).

[7]Andreasen, Fernandez-Villaverde, and Rubio-Ramirez (2018) also points to the increased volatility of price dispersion with steady state inflation and they use this fact together with price indexation to match the volatility of nominal term premium.

data. The key contributions of this article are thus, to offer both a warning about potential pitfalls of the Calvo setting and some guidance to the macroeconomic modelers for avoiding these pitfalls.

The rest of the paper proceeds as follows. Section 2 develops the main body of the paper. Section 2.1 documents in detail how simulated model moments are affected by the incorporation of trend inflation and discusses model devices that help remedy this situation. Section 2.2 is dedicated to a discussion of the channels that lead to high levels of price dispersion under the presence of trend inflation, and to the large inefficiencies they create. Section 2.3 uses model simulations to further develop an understanding of the behavior of price dispersion and studies its numerical properties. Section 3 concludes.

2. The baseline Rudebusch and Swanson model with trend inflation

Our example model, the Rudebusch and Swanson model with trend inflation, is in many aspects a standard model in the New Keynesian tradition. A continuum of firms operate under monopolistic competition and are subject to nominal rigidities à la Calvo. Households have preferences over consumption and labor –albeit in the form of Epstein-Zin preferences instead of the more conventional CRRA preferences. The central bank follows a Taylor rule, with a time-varying inflation target that is centered around a positive steady-state inflation level, instead of a zero trend inflation as in the original article.[8] Throughout the paper, Π_t denotes the gross inflation rate, defined as $\Pi_t = P_t/P_{t-1}$; lower case variable π_t instead denotes the (annualized) net inflation rate in percent, $\pi_t = 100\log(\Pi_t^4)$.

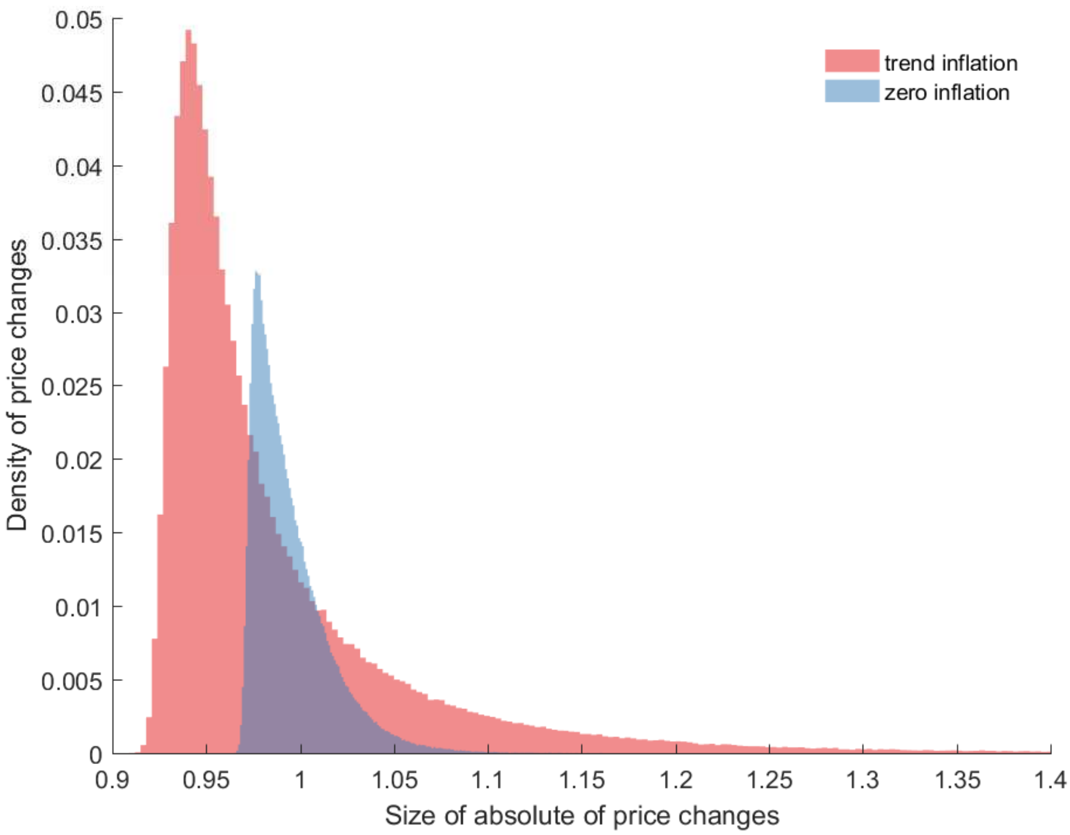
2.1. Model moments

Table 1 illustrates that relaxing the assumption of zero trend inflation ($\bar{\pi} = 0$), and, instead, allowing for positive trend inflation ($\bar{\pi} > 0$) in the RS model, produces unreasonable, largely inflated macro and finance unconditional second moments. The mechanism that accelerates the model dynamics is closely linked to the distribution of prices in the model economy. Figure 1 shows the simulated distribution of price changes. The

[8] To keep the exposition of this comment concise, we refrain from a section that describes the model in detail. We refer the interested reader to Appendix A.1 for a sketch of the model, or to the original article.

main effect of trend inflation is to make large price changes more likely because the subset of firms that can change the price needs to react to the positive trend in inflation when changing the price. Some firms with relatively low prices fixed far in the past will have to make large price changes to compensate for the rise in the price level that took place over time due to trend inflation. The distribution of prices can be described succinctly by the measure of the price dispersion, S_t, defined as

Figure 1: Simulated Distribution of Price Changes



Note: The shaded areas plot the simulated distributions of the size of absolute price changes, $\frac{P_t^*}{P_t}$, implied by the Calvo pricing model with (red) and without (blue) trend inflation.

$$
\begin{aligned}
S_t^{\frac{1}{1-\theta}} &\equiv \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\varepsilon}{1-\theta}} di \\
&= (1-\zeta)\,(p_t^*)^{\frac{-\varepsilon}{1-\theta}} + \zeta\,(\Pi_t)^{\frac{\varepsilon}{1-\theta}}\, S_{t-1}^{\frac{1}{1-\theta}};
\end{aligned} \tag{1}
$$

where ζ is the Calvo parameter (the per-period probability that the price cannot be changed), $1-\theta$ is the labor income share, and ε is the elasticity of substitution between varieties. Given the definition in equation (1), price dispersion, S_t, is bounded by 1 from below, or, equivalently, S_t^{-1} is bounded by 1 from above which means that when

$S_t = 1$ all firms have the same prices in the economy.

Table 1: Empirical and Model-Based Unconditional Moments

Unconditional Moment	US data 1961-2007	RS1 $\bar{\pi} = 0\%$	RS2 $\bar{\pi} = 1.6\%$	RS3 $A_t N_t^{1-\theta}$	RS4 $A_t K_t^{\theta} N_t^{1-\theta}$	RS5 $\pi_t^* = \bar{\pi}^*\%$
SD(dC)	2.69	0.72	8.29	7.83	5.89	7.42
SD(C)	0.83	0.88	12.88	11.13	447.44	9.60
SD(N)	1.71	2.51	38.16	30.99	482.68	27.27
Mean(i)	5.72	3.06	0.46	4.12	-787.40	1.98
SD(i)	2.71	3.41	49.39	41.39	2212.94	34.86
Mean(π)	3.50	-0.54	-2.12	1.42	-791.14	-0.66
SD(π)	2.52	3.01	40.84	36.47	2211.99	29.83
SD($i^{(40)}$)	2.41	2.33	31.25	29.62	2215.71	23.67
Mean($NTP^{(40)}$)	1.06	0.91	2.50	3.41	0.55	3.23
SD($NTP^{(40)}$)	0.54	0.42	7.21	6.63	5.94	6.27
Mean($R^{(40)} - R$)	1.43	0.88	2.72	3.24	0.98	2.90
SD($R^{(40)} - R$)	1.33	1.59	26.57	21.95	36.31	19.98
Mean(S^{-1})	< 1.00	0.99 [0.83,1.07]	1.05 [0,56]	1.01 [0.83,1.07]	1014.74 [0,7e^5]	1.01 [0,47]

Note: All variables are quarterly values expressed in percent. Inflation, interest rates and the term premium are expressed at an annual rate. The red colored numbers represent values of the inverse price dispersion that violate the economically feasible range, as S^{-1} is bounded from above by one. The interval indicated below row 'Mean(S^{-1})' reports the range (minimum and maximum values) of S^{-1} observed over the simulation. ***RS1*** *is the original RS model which has following features: fixed capital $Y_t = A_t \bar{K}^{\theta} N_t^{1-\theta}$, time-varying inflation target, π_t^*, zero trend inflation, $\bar{\pi} = 0\%$.* ***RS2*** *is RS1 with positive trend inflation $\bar{\pi} = 1.6$.* ***RS3*** *is RS1 with trend inflation $\bar{\pi} = 1.6$ and a labor-only-DRS production function, $Y_t = A_t N_t^{1-\theta}$.* ***RS4*** *is RS1 with trend inflation $\bar{\pi} = 1.6$ and variable capital $Y_t = A_t K_t^{\theta} N_t^{1-\theta}$.* ***RS5*** *is RS1 with trend inflation $\bar{\pi} = 1.6$ and a constant inflation target in Taylor rule, $\pi_t^* = \bar{\pi}^*$.*

The first column of table 1 reports targeted empirical moments. The subsequent columns are model-based unconditional moments, calculated from third-order approximated and pruned model simulations of several model versions of the RS model. Column RS1 reports simulated moments from the original baseline RS model with zero trend inflation, using the RS best fit calibration from Table 3 of their paper[9]. Column RS2 reports results for the RS model with an annualized steady-state inflation of 1.6%.[10]

Even this very moderate level of trend inflation inflates the model moments, both macro and finance, to unrealistic values. Whereas, under the assumption of zero trend inflation, the mean and standard deviation of S_t^{-1} stays in the economically justifiable range

[9]The model calibration is summarized in Table 4.

[10]Note that this is an only very modest assumed level of annualized trend inflation. Empirically, the observed value of annualized trend inflation lies well above 2% for most of the sample periods since the second world war. However, we confirm Ascari and Ropele's (2009) result that at a rate higher than 1.6% the model solution becomes indeterminate for the empirically relevant calibration of the Taylor rule. Note that the Calvo pricing mechanism imposes an upper bound on inflation, which in our model setup is above 16% of annualized inflation. Nevertheless, much of the explosive dynamics we observe after introducing positive trend inflation can be attributed to the fact that our approximation method does not accurately capture this upper bound on inflation as conjectured by Andreasen and Kronborg (2017).

(a value of 0.99 can be interpreted as an output loss of 1 percent due to price dispersion), with trend inflation (column RS2) the mean of the inverse price dispersion becomes economically unfeasible. Also a large standard deviation and the wide range over which values of S_t^{-1} are observed in a simulation (reported in the squared brackets below the values of column 'Mean(S^{-1})') documents that periods where almost all output is lost due to price dispersion are frequent. Column RS3 reports moments for a model version where the feature of fixed capital is removed and replaced by a labor-only DRS production function; column RS4 is a version when capital is allowed to be variable, as in a standard Cobb-Douglas production function. Column RS5 relaxes RS's assumption of a time-varying inflation target and replaces it with a fixed target (as is more common in standard New Keynesian (NK) models). In all cases we observe that the problems of counterfactually high levels of price dispersion persist. In Appendix B we develop a set of results for a version of the standard CGG NK model that mirrors the findings just described, documenting that poor model performance is not specific to the asset pricing features of our example model. [11]

After we identified an unreasonable behavior of price dispersion as the main culprit in the poor performance of models with a Calvo pricing mechanism with trend inflation, we now turn to a number of candidates of model specifications that restore the model's moments fit, which are reported in Table 2. Table 2 presents simulated moments from versions of the trend-inflation-augmented-RS model, where $i)$ the assumption about how prices are set in the economy is modified to a setting with Rotemberg adjustment costs (instead of Calvo pricing), where $ii)$ we use a linear production function instead of decreasing return to scale (DRS), or where $iii)$ we remove the effects of trend inflation by inflation indexation to either steady-state inflation or last period inflation. (As before, Appendix B shows a set of parallel results for the CGG New Keynesian model.) The model features just discussed, which fix the problems with exploding moments documented in Table 1, have a common mechanism: they mitigate the dispersion of prices in the economy. The following subsection describes the mechanisms at play in more depth.

[11] To be precise, while the channels (to be described in detail in section 2.2 below) that drive up price dispersion are present at all times, whether or not they lead to the aforementioned problems also in the NK model is a quantitative matter. For example, we describe a model version of the NK model with difference-stationary technology shocks, i.e. shocks to the economy's growth rate. With moderate but persistent shocks, we can demonstrate that the same set of problems as in RS also arises in the NK model. The intuition is clear: with persistent shocks the dispersion of prices across the economy will increase because firms which set their prices infrequently face very different economic conditions. When we employ a parameterization in which shocks are less persistent and have a milder impact on the real economy, as well as for a CGG version with trend-stationary shocks, we find that model dynamics stay within the standard range.

Table 2: Empirical and Model-Based Unconditional Moments

Unconditional Moment	RS2 $\bar{\pi} = 1.6\%$	RS6 Rotemberg	RS7 $Y_t = A_t N_t$	RS8 $\iota = 0$	RS9 $\iota = 1$
SD(dC)	8.29	0.43	0.45	0.71	0.49
SD(C)	12.88	0.49	0.53	0.89	0.68
SD(N)	38.16	1.45	1.39	2.50	1.85
Mean(i)	0.46	3.16	4.80	5.73	4.83
SD(i)	49.39	2.09	2.46	3.43	3.07
Mean(π)	-2.12	-0.48	1.05	2.22	1.42
SD(π)	40.84	2.14	2.33	2.98	2.58
SD($i^{(40)}$)	31.25	1.54	1.54	2.37	1.84
Mean($NTP^{(40)}$)	2.50	0.83	0.64	1.08	1.23
SD($NTP^{(40)}$)	7.21	0.36	0.10	0.55	0.03
Mean($R^{(40)} - R$)	2.72	0.84	0.61	1.11	1.27
SD($R^{(40)} - R$)	26.57	1.03	1.13	1.61	1.59
Mean(S^{-1})	1.05	0.00	1.00	0.99	1.00
SD(S^{-1})	0.82	0.00	0.00	0.01	0.00

Note: All variables are quarterly values expressed in percent. Inflation, interest rates and the term premium are expressed at an annual rate. Unlike in Table 1, there are no observations of the inverse price dispersion in violation of the economically feasible range. ***RS2****: equal to RS2 from Table 1.* ***RS6****: as in RS2, but with Rotemberg adjustment costs instead of Calvo pricing.* ***RS7****: as in RS2, but with a labor-only-CRS production function, $Y_t = A_t N_t$.* ***RS8****: as in RS2, but with indexation to steady-state inflation ($\iota = 0$).* ***RS9****: as in RS2, but with indexation to last-period inflation ($\iota = 1$).*

2.2. Trend inflation and price dispersion – channels

A well-known feature of the Calvo assumption that in each period only a fraction of firms is allowed to re-set their prices optimally is that firms with many different prices co-exist in the economy, captured by the measure of price dispersion, S_t, equation ((1)). As first brought to light in a paper by Ascari (2004), and further contributions by the same author that are summarized in Ascari and Sbordone (2014), price dispersion raises the resource cost of production by introducing a wedge between aggregate output and the amount of inputs[12] needed to produce this level of output, $Y_t = S_t^{-1} A_t K_t^{\theta} N_t^{1-\theta}$. This wedge becomes significantly amplified in the case of trend inflation. Trend inflation adds a drift into the evolution of prices and, thus, drives the distribution of prices further apart from the average price index P_t. To better understand the mechanism at play, we lay out two channels through which trend inflation has a key influence on price dispersion and the dynamics of real economic variables: i) the marginal-cost channel

[12]Ascari and Sbordone (2014) discuss the steady-state implications of trend inflation, whereas our focus is more on the dynamics, which is crucial for asset pricing.

and ii) a trend-inflation markup channel[13].

2.2.1 The marginal-cost channel

Let, in the following, variables carrying an asterisk denote prices and quantities of a firm that, in period t, is allowed to re-set its price optimally . Let variables without asterisk denote aggregate economy-wide variables, that include firms that are not allowed to re-set their price in the current period and are stuck with prices from the past. Appendix A shows formally that the dispersion of prices in the economy with positive trend inflation leads to a situation where the firm that, at t, is allowed to re-set its price optimally chooses to produce less than the average firm, so that $Y_t^* < Y_t$ (equation (37)). The average firm will therefore hire more labor units, $N_t > N_t^*$, and under DRS face higher marginal costs, $MC_t > MC_t^*$. The wedge between the quantities of the price re-setting and the average firm (denoted $\phi_{\{n,mc,y\},t}$) generally depends on the ratio of two price indexes: the price adjustment gap, P_t^*/P_t, and price dispersion, S_t, (as defined in equation (1)).

For the DRS case (as in the RS model) the ratio of labor demands between the firm re-setting its price at time t, N_t^*, and the average firm, N_t, is given by,

$$N_t^* = \phi_{n,t} N_t \qquad \text{where} \qquad \phi_{n,t} = \frac{\left(\frac{P_t^*}{P_t}\right)^{-\frac{\epsilon}{1-\theta}}}{\left[\int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{\frac{-\epsilon}{1-\theta}} dj\right]}. \tag{2}$$

Firms that are not able to reset their prices hire an inefficient amount of labor where $\phi_{n,t}$ can be interpreted as a measure of labor market inefficiency[14].

Proposition 2 in Appendix A shows that in a setting *without trend inflation* ($\bar{\pi} = 0$), the ratio of the price adjustment gap to price dispersion will be smaller than one, so that $\phi_{n,t} < 1$, in states of the economy with positive inflation realizations, $\pi_t > 0$, and bigger than one, so that $\phi_{n,t} > 1$, in states in which $\pi_t < 0$. This means that unless for the case of $\pi_t = 0$, and where all firms charge the same price, the average firm that cannot adjust its price will hire more (less) labor than optimal, $N_t > N_t^*$ ($N_t < N_t^*$), depending on

[13] Our decomposition is somewhat different compared to other contributions in the literature, where the focus is on a trend-inflation markup channel. For example, Ascari and Sbordone (2014) decompose the markup, ϕ_t, into a price adjustment gap, P_t^*/P_t and P_t^*/MC_t to study the implications of trend inflation for the model's deterministic steady-state. Our discussion of the marginal-cost channel is in this sense novel.

[14] It should be noted that already Ascari (2004) discusses a related effect by looking at the production function, and pointing to the fact that the relationship between employment and output is proportional to the price adjustment gap.

states of nature with $\pi_t > 0$ ($\pi_t < 0$). In the case of *positive trend inflation*, ($\bar{\pi} > 0$), the probability of realizations of deflationary states of the world decreases, because current inflation needs to fall not only below its steady-state value of $\bar{\pi} > 0$ but below zero. Equivalently, the likelihood of observing states of nature where $\pi_t > 0$ increases. For this reason, the value of $\phi_{n,t}$ will be less than one, $\phi_{n,t} < 1$ for most of the states of the world and accordingly also moves the average value $\phi_{n,t}$ below one. Positive trend inflation thus amplifies the inefficiency of the labor market.

In the DRS case, the effect of price dispersion on the real economy is magnified by the fact that the marginal costs of the average firm will be higher than the marginal costs of the price re-setting firm. Appendix A shows that,

$$MC_t^* = \phi_{mc,t} MC_t \qquad \text{where} \qquad \phi_{mc,t} = \frac{\left(\frac{P_t^*}{P_t}\right)^{-\frac{\theta\epsilon}{1-\theta}}}{S_t^{\frac{1}{1-\theta}}}. \tag{3}$$

Proposition 2 also implies that the mean of $\phi_{mc,t}$ will be less than one with trend inflation, $\phi_{mc,t} < 1$. The quantitative impact of the higher cost of production in a stochastic steady-state on the economic dynamics is substantial. The explanation is straightforward. The average firm needs to employ more factor inputs to meet the higher demand for its goods (given by its lower prices), and, as its moves along the concave production function to the right, the marginal costs rise with the level of production. The fact that the average firm will produce at a higher marginal cost than optimal at time t adds an additional inefficiency in the production and amplifies the real costs of price dispersion in the economy.

In the case of constant returns to scale[15], all firms face the same marginal costs (equation (34)), and this channel is muted. However, in case of CRS, the average firm will still produce more output and thus employ more factor inputs[16] (equation (37) and 2).

2.2.2 Trend-inflation markup channel

The presence of trend inflation leads firms to set their price at an additional markup over (current and future expected) marginal costs, which we call the trend-inflation markup: a markup implied by sticky prices and elevated by trend inflation that occurs over and above the traditional markup from monopolistic competition. Trend inflation enters the firm price decision problem, and therefore the first order condition for the optimal price

[15] Especially in case of a linear production function, $\theta = 0$.

[16] In the model with capital the wedge between capital hired by the average and the price re-setting firm will further amplify the effects of price dispersion.

represents another important channel. The price re-setting firm is forward-looking, it can foresee trend inflation and will therefore, on average, set its price above the aggregate price level (which includes non-resetting firms' prices from the past), $P_t^* > P_t$. It is because the optimal price has to equate the present value of future marginal revenues with marginal costs,

$$\sum_{k=0}^{\infty} \zeta^k E_t Q_{t,t+k} \Pi_{t+k}^{\epsilon-1} Y_{t+k} \left(\frac{P_t^*}{P_t}\right)^{1+\frac{\varepsilon\theta}{1-\theta}} = \frac{\epsilon}{\epsilon-1} \sum_{k=0}^{\infty} \zeta^k E_t Q_{t,t+k} \Pi_{t+k}^{\frac{\epsilon}{1-\theta}} Y_{t+k} MC_{t+k}^r(j), \quad (4)$$

The trend growth in prices increases both the firms' costs of production and the revenues from the sold output. Nevertheless, nominal marginal costs (the expression in the infinite sum on the right hand side of equation (4)) grow at a faster rate than nominal revenues (the left hand side of equation (4)). So, to keep the equality of marginal revenues with marginal cost in present value terms, the price setting firm must set P_t^* above P_t.[17] The difference between the rate of growth in marginal cost and marginal revenue shapes the firm's markup over (present and future) marginal costs. Equation (5) defines the price adjustment gap that depends on the weighted average of the firm's current and expected future real marginal costs.

$$\left(\frac{P_t^*}{P_t}\right)^{1+\frac{\varepsilon\theta}{1-\theta}} = \frac{\epsilon}{\epsilon-1} E_t \sum_{k=0}^{\infty} \phi_{t+k} MC_{t+k}(j) \qquad \text{where} \qquad \phi_{t+k} = \frac{m_{t+k}\Pi_{t+k}^{\frac{\epsilon}{1-\theta}}}{\sum_{k=0}^{\infty} m_{t+k}\Pi_{t+k}^{\epsilon-1}}, \quad (5)$$

where $m_{t+k} = \zeta^k E_t Q_{t,t+k} Y_{t+k}$. Ascari and Sbordone (2014) show that the mark-up, ϕ_t, increases with inflation[18]– and, thus, as trend inflation increases, the firm's trend-inflation markup amplifies the distortion implied by monopolistic competition.

The rise in ϕ_t means that firms put more weight on marginal costs far in the future compared to current marginal costs[19]. Future marginal costs are discounted by the model's implied yield curve with maturity k, where $Q_{t,t+k}(1/\Pi_{t+k})$ is the nominal price of the bond with maturity k. In the model with an upward sloping yield curve and high inflation risks, firms will discount the future relatively more.

[17] Note that trend-inflation markup channel is enforced by the markup channel through the parameter θ which further widens the gap between costs and revenues. Thus, strictly speaking there is a third interaction channel.

[18] As Π goes up, the numerator grows faster – at rate $\Pi_{t+k}^{\frac{\epsilon}{1-\theta}}$ – than the denominator –which grows by $\Pi_t^{\epsilon-1}$

[19] Ascari and Sbordone (2014) shows that overly forward looking agents de-anchor inflation expectations and decrease the determinacy region. This fact also applies to our model as the model solution is indeterminate for $\bar{\pi} > 1.6\%$.

A decrease in the wedge between marginal costs can mitigate these channels, which can be done by introducing a (full) inflation indexation. Another option is to increase the monopolistic mark-up (decrease ϵ): having a larger mark-up allows the firm to accommodate bigger deviations from the optimal price. Note, also, that θ and ϵ increase the non-linearity of model equilibrium conditions, which, as we later show, substantially increases approximation errors.

2.3. Behavior and approximation accuracy of price dispersion

In what follows, we focus our analysis further on the dispersion of prices in the economy. As the model parameters of RS were calibrated to match moments for the case of $\bar{\pi} = 0$, it may be argued that the model might be not well calibrated. We first confirm that the patterns documented in Table 1 hold across a wide range of parameter values.

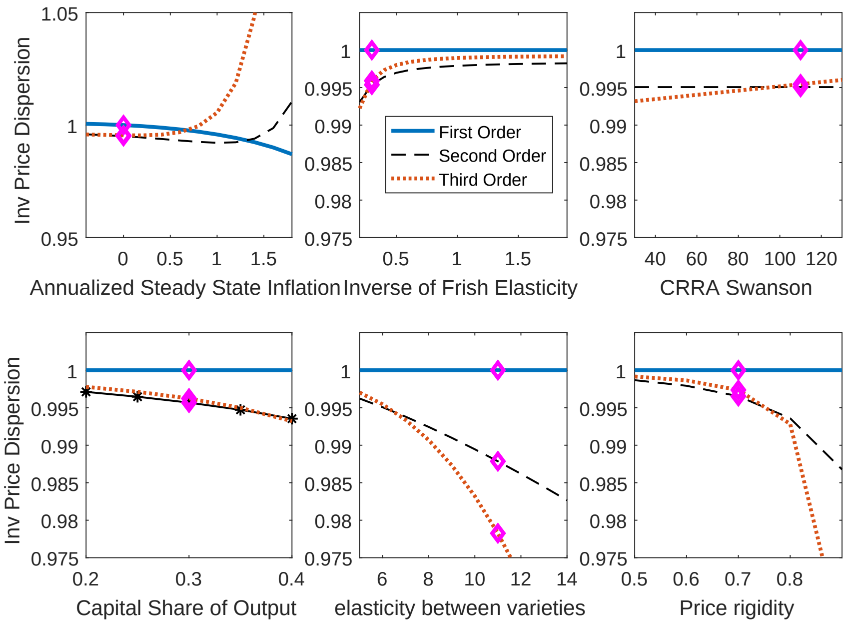
Figure 2 shows how mean of (the inverse) price dispersion changes over different ranges of parameter values and orders of approximation. The first set of panels shows the sensitivity of the mean simulated price dispersion to changes in key model parameters for the case of zero trend inflation, for different orders of approximation (first, second, third-order approximations). Pink diamonds reflect the case of the 'RS Table 3'-baseline parameterization. Whereas the mean simulated price dispersion is affected strongly by varying the trend inflation (panel 1), including pushing S^{-1} to an infeasible region[20], varying other model parameters does not affect the simulated mean price dispersion drastically (and never pushes S^{-1} to an infeasible region). Other than variations in trend inflation, only regions of relatively high elasticities of substitution or high price rigidities lead to large costs from price dispersion (of, e.g. more than 1%, reflected in S^{-1} falling below 0.99). The second set of panels presents comparable figures for the case of positive trend inflation. Pink diamonds reflect the 'RS Table 3'-baseline parameterization, apart for steady-state inflation, which now is $\bar{\pi} = 1\%$. Since the accuracy of the mean price dispersion is already somewhat compromised at $\bar{\pi} = 1\%$, regions of relatively high elasticities of substitution or high price rigidities quickly lead to problems (flat lines in the last two reported panels represent cases with indeterminate solutions). Variations in other key parameters continue to leave mean price dispersion mostly unaffected.

To further examine the role of price dispersion in generating explosive dynamics we look into the numerical accuracy of the approximation to the price dispersion equation (1).

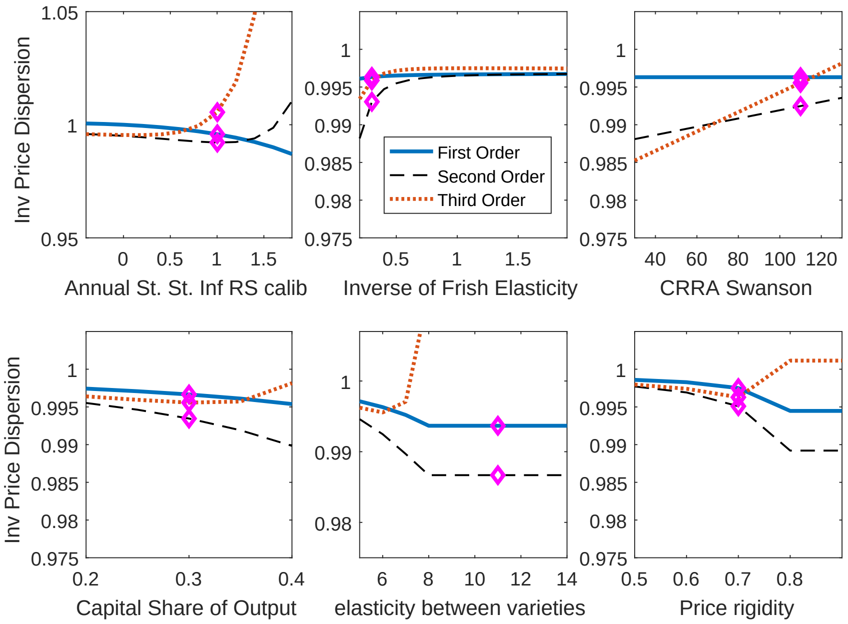
[20] S^{-1} is bounded from above by one. See Proposition 1

Figure 2: Parameters sensitivity in the RS (2012) model

Parameter sensitivity of RS (2012) model, case of zero trend inflation, $\bar{\pi} = 0\%$

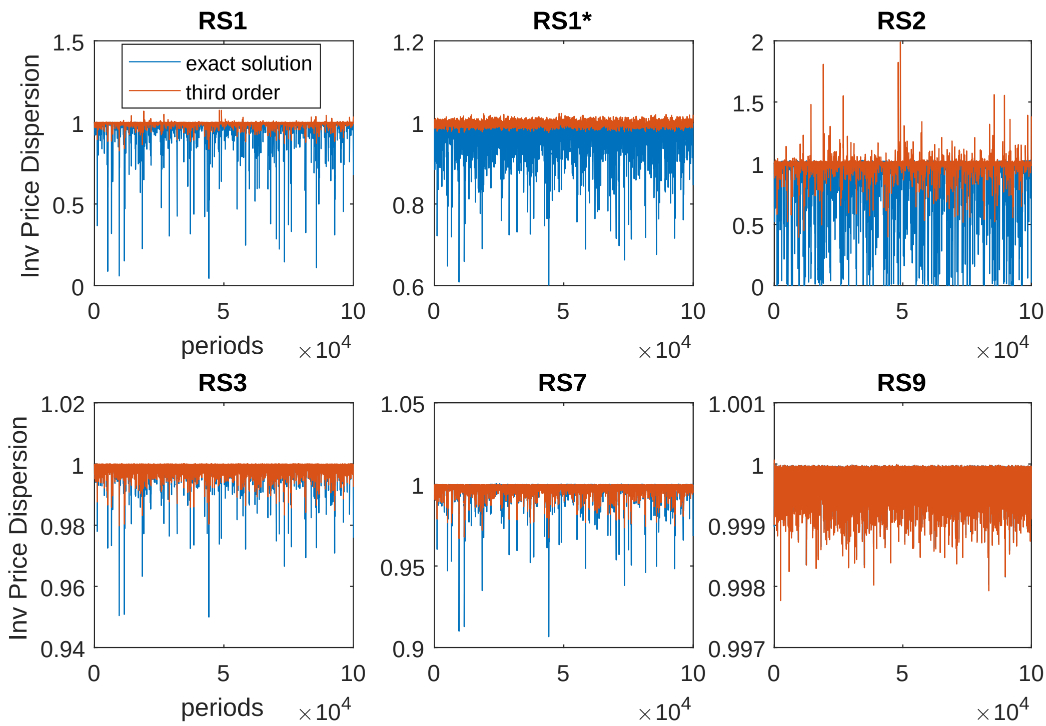


Parameter sensitivity of RS (2012) model, case of positive trend inflation, $\bar{\pi} = 1\%$



Note: The first set of panels shows the sensitivity of the mean simulated price dispersion to changes in key model parameters for the case of zero trend inflation, for different orders of approximation (first, second and third order approximations). Pink diamonds reflect the case of the 'RS Table 3'-baseline parameterization. The second set of panels presents analogous figures for the case of positive trend inflation. Flat lines in the last two reported panels represent cases with indeterminate solutions.

Figure 3: Approximation Errors for Price Dispersion



Note: The panels contrast simulated paths for price dispersion, as computed from a third-order approximation of the model with the 'exact' behavior for price dispersion, using equation (6), conditioning on the simulated time path of Π from the third-order-approximated model. ***RS1****: original RS model, with the following features: fixed capital $Y_t = A_t \bar{K}^{\theta} N_t^{1-\theta}$, time-varying inflation target, π_t^*, zero trend inflation, $\bar{\pi} = 0\%$.* ***RS1****: as in RS1, but approximated only up to the first order.* ***RS2****: as in RS1, but with positive trend inflation of $\bar{\pi} = 1\%$.* ***RS3****: as in RS1, but with trend inflation of $\bar{\pi} = 1\%$ and a labor-only-DRS production function, $Y_t = A_t N_t^{1-\theta}$.* ***RS7****: as in RS1, but with trend inflation of $\bar{\pi} = 1\%$ and with a labor-only-CRS production function, $Y_t = A_t N_t$.* ***RS9****: as in RS1, but with trend inflation of $\bar{\pi} = 1\%$ and with indexation to last-period inflation ($\iota = 1$).*

Alongside the more rigorous study of Andreasen and Kronborg (2017) on numerical accuracy of approximation methods we calculate a more accurate measure of price dispersion, noting that price dispersion can be written recursively as

$$S_t^{\frac{1}{1-\theta}} = (1-\zeta)\left[\frac{1-\zeta\,(\Pi_t)^{\epsilon-1}}{1-\zeta}\right]^{\frac{\epsilon}{(\epsilon-1)(1-\theta)}} + \zeta(\Pi_t)^{\frac{\epsilon}{1-\theta}} S_{t-1}^{\frac{1}{1-\theta}}. \tag{6}$$

We proceed as follows.[21] First, we use an initial value S_{t-1} from the approximated model as a starting point. Second, we iterate the equation forward to get an exact solution conditional on the model-approximated time path of Π_t. Third, we compare this more exact measure of price dispersion with its counterpart from the third- order approximation[22].

[21] We very much thank Larry Christiano for suggesting to look at the problem in this way.

[22] Andreasen and Kronborg (2017) shows that although the conditioning on inflation delivers somewhat different solution compared to the use of more accurate projection methods, the approximation errors of our more exact measure should be small. For this reason, the conditioning on inflation should not harm

The panels in Figure 3 contrast simulated paths for price dispersion, as computed from a third-order approximation of the model with the 'exact' behavior for price dispersion, using equation (6) of the main text, for several model versionS. As can be seen, the third order approximation a) deviates sharply from the path of price dispersion using the exact formula, and b) includes many infeasible realizations of $S^{-1} > 1$. The problems diminish or disappear when adopting one of the proposed fixes documented in section 2.1.

Subpanel 'RS1' of Figure 3 stresses this finding by showing that the approximation of price dispersion is poor even for the original RS model[23] as the deviations between the third-order and 'exact' solution are large. Perturbation methods do an even poorer job in the case of positive trend inflation. In addition, in the case of positive inflation the third order approximation generates state of the worlds which are economically infeasible as S_t^{-1} exceeds one, which means that more resources are spent than produced, $Y_t < C_t + I_t + G_t$. Subpanel 'RS1*' in the second row shows that a first-order approximation delivers smaller approximation errors.

The approximation errors for the cases of indexation to past inflation and constant return to scale in labor are negligible. The RS model with positive steady-state inflation and indexation delivers both small price dispersion and negligible approximation errors, as can be observed by the almost complete overlay of the two simulated series. However, it should be noted that in the case of the linear production function, the more exact measure of price distortion is still large. There are states of the world when the price dispersion implies an almost 10% quarterly output loss, which is at odds with empirical evidence (see, for example, Nakamura, Steinsson, Sun, and Villar (2016)). Andreasen and Kronborg (2017) conjectures that this explosive dynamics in price dispersion come from the price-inflation spiral generated by the fact that the perturbation methods up to third order fails to account for an upper bound on inflation.

3. Conclusion

This note emphasizes that an attempt to realign the current macro-finance workhorse modeling framework of RS with recent empirical evidence should include incorporating positive trend inflation into such a framework. We document that pricing assets in models that are based on the Calvo price mechanism can lead to extremely counterfactual model dynamics, once trend inflation is present; we then propose a number of direc-

our argument.

[23]Note that the original Rudebusch and Swanson (2012) results are sensitive to the seed of random number generator even for very long simulations.

tions to overcome such complications. This way, we contribute to providing guidance along the path of finding a new, empirically well-motivated and consistent modeling framework.

A. Rudebusch and Swanson (RS) Model

This appendix gives a brief overview of the example model of Rudebusch and Swanson (2012).

A.1. Model sketch, RS model

A.1.1 Households

The description of the households and firms' problems below closely follows RS.

The household maximizes the continuation value of its utility (V), which is of the Epstein-Zin form and follows the specification of RS:

$$V_t = \begin{cases} U(C_t, N_t) + \beta \left[E_t V_{t+1}^{1-\alpha}\right]^{\frac{1}{1-\alpha}} & \text{if } U(C_t, N_t) \geq 0, \\ U(C_t, N_t) - \beta \left[E_t(-V_{t+1})^{1-\alpha}\right]^{\frac{1}{1-\alpha}} & \text{if } U(C_t, N_t) < 0. \end{cases} \tag{7}$$

The households' problem is subject to the flow budget constraint:

$$B_t + P_t C_t = W_t N_t + D_t + R_{t-1} B_{t-1}. \tag{8}$$

In equation (7), β is the discount factor. Utility (U) at period t is derived from consumption (C_t) and leisure $(1 - N_t)$. E_t denotes expectations conditional on information available at time t. As the time endowment is normalized to one, leisure time $(1 - N_t)$ is what remains after spending some time working (N_t). $W_t N_t$ is labor income, R_t is the return on the one-period nominal bond, B_t, D_t is dividend income.

To be consistent with balanced growth, RS impose the following functional form on U:

$$U(C_t, N_t) = \frac{C_t^{1-\varphi}}{1-\varphi} + \chi_0 Z_t^{1-\varphi} \frac{(1-N_t)^{1-\chi}}{1-\chi}, \quad \varphi, \chi > 0, \tag{9}$$

where Z_t is an aggregate productivity trend, and φ, χ, $\chi_0 > 0$. The intertemporal elasticity of substitution (IES) is $1/\varphi$, and the Frisch labor supply elasticity is given by $(1 - \overline{N})/\chi\overline{N}$, where $\overline{N}$ is the steady state level of hours worked.

A.1.2 Firms

Final good firms operate under perfect competition with the objective to minimize expenditures subject to the aggregate price level $P_t = \left[\int_0^1 P_t^{1-\epsilon}(i)(di)\right]^{\frac{1}{1-\epsilon}}$, where $P_t(i)$ is the price of intermediate good produced by firm i, using the technology $Y_t = \left[\int_0^1 Y_t^{\frac{\epsilon-1}{\epsilon}}(i)di\right]^{\frac{\epsilon}{\epsilon-1}}$. Final good firms aggregate the continuum of intermediate goods i on the interval $i \in [0,1]$ into a single final good. Parameter ϵ determines the elasticity of substitution between goods variety. The cost-minimisation problem of final good firms delivers demand schedules for intermediary goods of the form:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} Y_t. \tag{10}$$

A continuum of intermediate firms operates in the economy. Intermediate firm i produces according to a Cobb-Douglas production function, where θ denotes the capital share. Aggregation across firms, yields:

$$S_t Y_t = A_t \bar{K}^{\theta} (Z_t N_t)^{1-\theta}. \tag{11}$$

$\bar{K}$ refers to the fact that firms have fixed capital[24] and S_t is the cross-sectional price dispersion. In equation (11) technology follows the autoregressive process:

$$\log A_t = \rho_A \log A_{t-1} + \sigma_A \epsilon_t^A, \tag{12}$$

where ϵ_t^A is an independently and identically distributed (i.i.d.) shock with zero mean and constant variance.

Intermediate firms maximize the present value of future profits facing Calvo contracts by choosing price, $P_t(i)$,

$$E_t \left\{ \sum_{k=0}^{\infty} \zeta^k Q_{t,t+k} \frac{P_t}{P_{t+k}} \left[P_t(i) Y_{t+k}(i) - W_{t+k} N_{t+k}(i)\right] \right\}, \tag{13}$$

where $Q_{t,t+j}$ is the real stochastic discount factor from period t to $t+k$. The term $W_{t+j}N_{t+j}(i)$ represents the cost of labor. The optimal price is a weighted average of current and future expected nominal marginal costs,

[24] Firm-specific capital can be interpreted as a model with endogenous investment that features high adjustment costs in investment.

$$P_t(i) = \frac{\epsilon}{\epsilon - 1} \sum_{k=0}^{\infty} \Upsilon_{t+k} MC_{t+k}(i), \tag{14}$$

Where $\Upsilon_{t+k} = \frac{E_t \zeta^k Q_{t,t+k} \frac{P_t}{P_{t+k}} Y_{t+k}(i)}{E_t \sum_{k=0}^{\infty} \zeta^k Q_{t,t+k} \frac{P_t}{P_{t+k}} Y_{t+k}(i)}$ is the time varying mark-up implied by price rigidity and $\frac{\epsilon}{\epsilon-1}$ is the mark-up implied by monopolistic competition.

Average real marginal cost is defined as

$$MC_t = \frac{1}{1-\theta} \left(\frac{W_t}{A_t} \right) \left(\frac{Y_t}{\bar{K} A_t} \right)^{\frac{\theta}{1-\theta}}. \tag{15}$$

A.1.3 Fiscal Policy and Monetary Policy.

Government spending follows the process:

$$\log(g_t/\bar{g}) = \rho_G \log(g_{t-1}/\bar{g}) + \varepsilon_t^G, \quad 0 < \rho_G < 1, \tag{16}$$

where $\bar{g}$ is the steady-state level of $g_t \equiv G_t/Z_t$, and ε_t^G is an i.i.d. shock with mean zero and variance σ_G^2.

The model is closed by a monetary policy rule:

$$4i_t = 4\rho_i i_{t-1} + (1-\rho_i) \left[4(\bar{i} - \bar{\pi}) + (\pi_t^{avg}) + \phi_\pi (4(\pi_t^{avg}) - (\pi_t^*)) + \phi_Y \left(\frac{\mu_t Y_t}{\bar{\mu} \bar{Y}} - 1 \right) \right], \tag{17}$$

where i_t is the (net) policy rate, $i_t = log(1+i_t)$, π_t^{avg} is a four-quarter moving average of (net) inflation (defined below), and Y_t^* is the trend level of output $\bar{Y} Z_t$ (where $\bar{Y}$ denotes the steady-state level of Y_t/Z_t). π_t^* is the target rate of inflation, and ε_t^i is an i.i.d. shock with mean zero and variance σ_i^2. ρ_i captures the motive for interest rate smoothing. The four-quarter moving average of inflation (π_t^{avg}) can be approximated by a geometric moving average of inflation:

$$\pi_t^{avg} = \theta_{\pi^{avg}} \pi_{t-1}^{avg} + (1 - \theta_{\pi^{avg}}) \pi_t, \tag{18}$$

where $\theta_{\pi^{avg}} = 0.7$ ensures that the geometric average of inflation has an effective duration of approximately four quarters. The inflation target π_t^* is time varying and driven

Table 3: System of model equations, Rudebusch Swanson model

(RS1):	$V_t = \frac{c_t^{1-\varphi}}{1-\varphi} + \chi_0 \frac{(1-N_t)^{1-\chi}}{1-\chi} + \beta(E_t[(V_{t+1}\mu_{t+1}^{1-\gamma})^{1-\alpha}])^{\frac{1}{1-\alpha}}$
(RS2):	$Q_{t-1,t} = \mu_t^{-\gamma} \left(\frac{(V_t \mu_t^{1-\gamma})}{[E_{t-1}(V_t\mu_t^{1-\gamma})^{1-\alpha}]^{\frac{1}{1-\alpha}}} \right)^{-\alpha} \left(\frac{c_t}{c_{t-1}} \right)^{-\varphi}$
(RS3):	$\chi_0(1-N_t)^{-\chi} c_t^{\varphi} = w_t$
(RS4):	$1 = \beta E_t \left\{ Q_{t,t+1} \frac{(1+i_t)}{\Pi_{t+1}} \right\}$
(RS5):	$(p_t^*)^{1+\frac{\theta\epsilon}{1-\theta}} = \frac{aux_{1t}}{aux_{2t}}$
(RS6):	$aux_{1t} = \frac{\epsilon}{\epsilon-1} mc_t y_t + \beta\zeta Q_{t,t+1} \Pi_{t+1}^{\frac{\epsilon}{1-\theta}} aux_{1t+1}$
(RS7):	$aux_{2t} = y_t + \beta\zeta Q_{t,t+1} \Pi_{t+1}^{\epsilon-1} aux_{2t+1}$
(RS8):	$S_t Y_t = A_t \bar{K}^{\theta} (N_t)^{1-\theta}$
(RS9):	$S_t^{\frac{1}{1-\theta}} = (1-\zeta)(p_t^*)^{\frac{-\epsilon}{1-\theta}} + \zeta(\Pi_t)^{\frac{\epsilon}{1-\theta}} S_{t-1}^{\frac{1}{1-\theta}}$
(RS10):	$\Pi_t^{1-\epsilon} = (1-\zeta)(p_t^* \Pi_t)^{1-\epsilon} + \zeta$
(RS11):	$MC_t = \frac{1}{1-\theta} \overline{K}^{\frac{\theta}{1-\theta}} \frac{W_t}{A_t} \left(\frac{y_t}{A_t} \right)^{\frac{\theta}{1-\theta}}$
(RS12):	$y_t = c_t + \overline{I} + g_t$
(RS13):	$4i_t = 4\rho_i i_{t-1} + (1-\rho_i) \left[4(\bar{i} - \bar{\pi}) + (\pi_t^{avg}) + \phi_\pi(4(\pi_t^{avg}) - (\pi_t^*)) + \phi_Y \left(\frac{\mu_t Y_t}{\bar{\mu}\bar{Y}} - 1 \right) \right]$
(RS14):	$\pi_t^* = (1-\rho_{\pi^*}) 4\pi_t^{avg} + \rho_{\pi^*}\pi_{t-1}^* + \zeta_{\pi^*}(4\pi_t^{avg} - \pi_t^*) + \sigma_{\pi^*}\varepsilon_{\pi^*,t}$
(RS15):	$\pi_t^{avg} = \theta_{\pi^{avg}} \pi_{t-1}^{avg} + (1-\theta_{\pi^{avg}}) \pi_t$
(RS16):	$\log A_t = \rho_A \log A_{t-1} + \sigma_A \varepsilon_{A,t}$
(RS17):	$\log(g_t/\bar{g}) = \rho_G \log(g_{t-1}/\bar{g}) + \varepsilon_t^G$

by following process,

$$\pi_t^* = (1-\rho_{\pi^*}) 4\pi_t^{avg} + \rho_{\pi^*}\pi_{t-1}^* + \zeta_{\pi^*}(4\pi_t^{avg} - \pi_t^*) + \sigma_{\pi^*}\varepsilon_{\pi^*,t}. \tag{19}$$

A.1.4 System of model equations

Table 3 summarizes the system of equations of the Rudebusch Swanson model in terms of stationary allocations and real (relative) prices (i.e., in term of detrended and deflated variables, denoted by lowercase variables) defined as $c_t = \frac{C_t}{Z_t}$, $y_t = \frac{Y_t}{Z_t}$, $\Pi_t = \frac{P_t}{P_{t-1}}$, $w_t = \frac{W_t}{P_t Z_t}$, $p_t^* = \frac{P_t^*(j)}{P_t}$, $mc_t(i) = \frac{MC_t(i)}{P_t}$, $y_t = \frac{Y_t}{Z_t}$, $\mu_t = \frac{Z_t}{Z_{t-1}}$. The best fit calibration of the RS model based on their Table 3 is summarized in Table 4. In this setting, model dynamics are driven by three types of shocks, stationary technology shocks, government spending shocks, and inflation target shocks (in particular, there are no trend productivity shocks, so that $\mu_t = \frac{Z_t}{Z_{t-1}} = \mu$ is constant).

A.2. Aggregation

Here we describe in detail the aggregation across the i-firms in case of decreasing return to scale and constant return to scale production function.

A.2.1 Aggregate Price Index

The aggregate price index $P_t = \left[\int_0^1 P_t^{1-\epsilon}(j)dj\right]^{\frac{1}{1-\epsilon}}$ can be written using the Calvo result as,

$$\frac{P_t^*}{P_t} = \left[\frac{1-\zeta\,(\Pi_t)^{\epsilon-1}}{1-\zeta}\right]^{\frac{1}{1-\epsilon}}, \tag{20}$$

A.2.2 Aggregation for DRS

The production function of intermediate firm i is given by $Y_t(i) = A_t K^\theta N_t^{1-\theta}(i)$. Using this, plug in for $Y_t(i)$ into the demand for variety i, equation 10, solve for $N_t(i)$ and integrate over all varieties i. Since the workers are all the same the aggregation of hours worked is $N_t = \int_0^1 N_t(i)di$. The aggregation delivers,

$$N_t = \left(\frac{Y_t}{A_t K^\theta}\right)^{\frac{1}{1-\theta}} \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\frac{\epsilon}{1-\theta}} di, \tag{21}$$

which can be re-written as

$$Y_t = S_t^{-1} A_t K_t^\theta N_t^{1-\theta}, \tag{22}$$

where the variable $S_t^{\frac{1}{1-\theta}} = \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\epsilon}{1-\theta}} di$ defines the price dispersion.

A.2.3 Resetting firm vs. aggregate quantities for DRS

The demand function for the firm resetting its price at time t, for the horizon k is given by,

$$Y_{t+k|t} = A_{t+k}\bar{K}^\theta N_{t+k|t}^{1-\theta} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon} Y_{t+k}, \tag{23}$$

where P_t^* is the optimal price of firm resetting its price at time t for the horizon k. The factor demand of the price re-setting firm, $N_{t+k|t}$ is,

$$N_{t+k|t} = \left(\left(\frac{P_t^*}{P_{t+k}} \right)^{-\epsilon} \frac{Y_{t+k}}{A_{t+k}\bar{K}^{\theta}} \right)^{\frac{1}{1-\theta}}. \tag{24}$$

The ratio of the price re-setting (equation (24)) and the aggregate firm's factor demands (21)) is given by

$$\frac{N_{t+k|t}}{N_{t+k}} = \frac{\left(\left(\frac{P_t^*}{P_{t+k}} \right)^{-\epsilon} \frac{Y_{t+k}}{A_{t+k}\bar{K}^{\theta}} \right)^{\frac{1}{1-\theta}}}{\left[\frac{Y_{t+k}S_{t+k}}{A_{t+k}K^{\theta}} \right]^{\frac{1}{1-\theta}}} = \frac{\left(\frac{P_t^*}{P_{t+k}} \right)^{-\frac{\epsilon}{1-\theta}}}{S_{t+k}^{\frac{1}{1-\theta}}} = \frac{\left(\frac{P_t^*}{P_{t+k}} \right)^{-\frac{\epsilon}{1-\theta}}}{\left[\int_0^1 \left(\frac{P_{t+k}(j)}{P_{t+k}} \right)^{\frac{-\epsilon}{1-\theta}} dj \right]}. \tag{25}$$

An analogous ratio can be derived for aggregate marginal cost and marginal costs of the price resetting firm. Marginal costs for the price resetting firm are,

$$MC_{t+k|t} = \frac{W_{t+k}}{(1-\theta)A_{t+k}K^{\theta}N_{t+k}^{-\theta}} \frac{N_{t+k}^{-\theta}}{N_{t+k|t}^{-\theta}}, \tag{26}$$

Aggregate marginal cost come from, $\frac{\partial W_t N_t}{\partial Y_t}$ and using $N_{t+k} = \left[\frac{Y_{t+k}}{A_{t+k}K^{\theta}} \right]^{\frac{1}{1-\theta}} S_{t+k}^{\frac{1}{1-\theta}}$. delivers,

$$\frac{MC_{t+k}}{S_{t+k}} = \frac{W_{t+k}}{(1-\theta)\left(A_{t+k}K^{\theta}N_{t+k}^{-\theta}\right)}. \tag{27}$$

Plugging equation (25) into (26) and rearranging delivers,

$$MC_{t+k|t} = MC_{t+k} \frac{\left(\frac{P_t^*}{P_{t+k}} \right)^{-\frac{\theta\epsilon}{1-\theta}}}{S_{t+k}^{\frac{1}{1-\theta}}} \tag{28}$$

A.2.4 Aggregation for CRS

The cost minimization problem is given by

$$\min_{N_t(i)} W_t N_t(i) + R_t^k K_t + MC_t^r(i) \left[Y_t(i) - A_t K_t(i)^{\theta} N_t^{1-\theta}(j) \right], \tag{29}$$

subject to the production function, $Y_t(i) = A_t K_t(i)^{\theta} N_t^{1-\theta}(i)$, where $MC_t(i)$ is the multiplier associated with the constraint.

The firm's demand for labor,

$$W_t = MC_t^r(i)(1-\theta)A_t K_t(i)^{\theta} N_t^{-\theta}, \tag{30}$$

The firm's demand for capital,

$$R_t^k = MC_t^r(i)A_t \theta K_t(i)^{\theta-1} N_t^{1-\theta}(i), \tag{31}$$

Plugging the factor demands into the definition of total costs, $TC_t(i) = W_t N_t(i) + R_t^k K_t(i)$ delivers,

$$TC_t(i) = [MC_t^r(i)]\, Y_t(i). \tag{32}$$

Marginal costs are defined as a change in total cost when output changes, $\frac{dTC_t(i)}{dY_t(i)} = MC_t^r(i)$, which shows that the Lagrange multiplier equals real marginal costs.

From equation (30) and equation (31) we get that,

$$\frac{1-\theta}{\theta} = \frac{W_t N_t(i)}{R_t^k K_t(i)}. \tag{33}$$

Since factor prices are common for all the firms, the ratio of $\frac{1-\theta}{\theta}\frac{R_t}{W_t} = \frac{N_t(i)}{K_t(i)}$ is the same for all firms. Plugging factor demands into production function delivers,

$$MC_t^r = \int_0^1 MC_t^r(i)di = \frac{\left(R_t^k\right)^{\theta} W_t^{1-\theta}}{A_t \theta^{\theta}(1-\theta)^{1-\theta}}, \tag{34}$$

A.2.5 Resetting firm vs. aggregate quantities for CRS

From the relationship $Y_{t+k|t} = A_{t+k} K_{t+k|t}^{\theta} N_{t+k|t}^{1-\theta} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon} Y_{t+k}$ we can derive,

$$N_{t+k|t} = \frac{\left(\frac{P_t^*}{P_{t+k}}\right)^{-\frac{\epsilon}{1-\theta}}}{\left[\int_0^1 \left(\frac{P_{t+k}(j)}{P_{t+k}}\right)^{\frac{-\epsilon}{1-\theta}} dj\right]} \left(\frac{K_{t+k}}{K_{t+k|t}}\right)^{\frac{\theta}{1-\theta}} N_{t+k}. \tag{35}$$

The ratio of capital demand equations for the price resetting firm and the aggregate firm delivers,

$$\frac{K_{t+k}}{K_{t+k|t}} = \frac{Y_{t+k}}{Y_{t+k|t}}, \tag{36}$$

Using the labor demand equations of price resetting and aggregate firms, together with equation (36), we get the relationship,

$$Y_{t+k|t} = \frac{\left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon}}{\left[\int_0^1 \left(\frac{P_{t+k}(j)}{P_{t+k}}\right)^{\frac{-\epsilon}{1-\theta}} dj\right]^{1-\theta}} Y_{t+k}, \tag{37}$$

Later we show that in case of positive inflation, $Y_{t+k|t} \leq Y_{t+k}$ in the case of CRS. Thus, as from above $\frac{K_{t+k}}{K_{t+k|t}} = \frac{Y_{t+k}}{Y_{t+k|t}}$, then $K_{t+k|t} \leq K_{t+k}$ and thus $N_{t+k|t} \leq N_{t+k}$.

A.3. Proofs and Propositions

Proposition 1 Price dispersion is bounded by one, $S_t \geq 1$.

Proof. The aggregate price index, $P_t = \left[\int_0^1 P_t^{1-\epsilon}(i)\right]^{\frac{1}{1-\epsilon}}$ divide by P_t is $1 = \left[\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$ Defining $v_{i,t} = \left(\frac{P_t(i)}{P_t}\right)^{1-\epsilon}$ we get that $\left[\int_0^1 v_{i,t}\right]^{\frac{1}{1-\epsilon}} = 1$. Writing price dispersion, $S_t = \left[\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\epsilon}{1-\theta}} dj\right]^{1-\theta}$, in terms of $v_{i,t}$, $v_{i,}^{\frac{-\epsilon}{1-\epsilon}\frac{1}{1-\theta}} = \left[\left(\frac{P_t(i)}{P_t}\right)^{1-\epsilon}\right]^{\frac{-\epsilon}{1-\epsilon}\frac{1}{1-\theta}}$ Thus, price dispersion can be written in terms of variable v as, $S_t^{\frac{1}{1-\theta}} = \int_0^1 v_{i,t}^{\frac{\epsilon}{\epsilon-1}\frac{1}{1-\theta}}$ And as $\frac{\epsilon}{\epsilon-1}\frac{1}{1-\theta} > 1$, Jensen's inequality implies that

$$1 = \left[\int_0^1 v_{i,t}\right]^{\frac{\epsilon}{\epsilon-1}\frac{1}{1-\theta}} \leq \int_0^1 v_{i,t}^{\frac{\epsilon}{\epsilon-1}\frac{1}{1-\theta}} = S_t^{\frac{1}{1-\theta}}. \tag{38}$$

$\square$

Proposition 2 The ratio of price indexes, $\phi_n = \frac{\left(\frac{P_t^*}{P_t}\right)^{-\frac{\epsilon}{1-\theta}}}{\left[\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\epsilon}{1-\theta}} dj\right]} \gtreqqless 1$, for

$$\phi_n \begin{cases} < 1 & \text{for } \bar{\pi} = 0 \quad \& \quad \hat{\pi}_t > 0, \\ & \text{for } \bar{\pi} > 0 \quad \& \quad \hat{\pi}_t > -\bar{\pi}, \\ = 1 & \text{for } \bar{P} = P_t^* = P_t(j) = P_t, \\ > 1 & \text{for } \bar{\pi} = 0 \quad \& \quad \hat{\pi}_t < 0, \\ & \text{for } \bar{\pi} > 0 \quad \& \quad \hat{\pi}_t < -\bar{\pi}, \end{cases} \tag{39}$$

where $\bar{P}$ is the deterministic steady state of price and $\hat{\pi}_t$ is deviation of inflation from its steady state.

Proof. The ratio $\phi_n < 1$ if $\left(\frac{P_t^*}{P_t}\right)^{-\frac{\epsilon}{1-\theta}} < \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\epsilon}{1-\theta}} di$. From the Proposition 1, $\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\epsilon}{1-\theta}} di \geq 1$. Thus, it must be true that if $\left(\frac{P_t^*}{P_t}\right)^{-\frac{\epsilon}{1-\theta}} \leq 1$ then $\phi_n \leq 1$. This will hold for all cases when $P_t^* \geq P_t$. Because $\frac{P_t^*}{P_t} = \left[\frac{1-\zeta(\Pi_t)^{\epsilon-1}}{1-\zeta}\right]^{\frac{1}{1-\epsilon}}$ (equation (20)) for $\Pi_t \geq 1$ it holds that $P_t^* \geq P_t$ In case of positive steady state inflation, $\bar{\pi}_t > 0$ the inflation deviation from its steady state can reach $\hat{\pi}_t > -\bar{\pi}$ for $\phi_n \leq 1$. □

A.4. Calibration

Table 4: Calibration of the RS table 3 (best fit) model

Symbol	Variable	Value
β	Discount factor	0.99
$CRRA$	Risk aversion	110
IES	Intertemporal elasticity	0.09
ϵ	Elasticity of substitution	6
$Frisch$	Frisch elasticity	0.28
ϕ_π	Response to inflation	0.53
ϕ_y	Response to output	0.93
ρ_i	i_t smoothing	0.73
ζ	Price adjustment	0.76
$\bar{G}/\bar{Y}$	Government spending on output	0.17
ρ_G	Autocorrelation Government spending shock	0.95
σ_G	Volatility of Government spending shock	0.004
ρ_A	Autocorrelation of TFP shock	0.95
σ_A	Volatility of TFP shock	0.005
$\theta_{\rho_{pi*}}$	Inflation target shock persistence	0.995
$\sigma_{\pi*}$	Volatility of inflation target shock	0.0007
$\zeta_{\pi*}$	Inflation target adjustment	0.003
θ	Capital share of output	1/3
$\bar{\Pi}$	Steady state inflation	1.004
δ	Capital depreciation	0.02

B. Basic New Keynesian (CGG) Model

This section of the appendix outlines the basic New Keynesian model and presents results analogous to the one in the main text. The model closely follows the sticky price model of Clarida, Gali, and Gertler (1999), with two exceptions: one, we use a production function that is assumed to be of the DRS-labor-only type as our baseline, as in the RS model. Two, we assume that productivity shocks are difference-stationary (in the case of trend-stationary shocks the channels leading to high levels and poor approximation of price dispersion are quantitatively inconsequential).

Otherwise, the model features are standard, firms are monopolistically competitive, face nominal rigidities à la Calvo, and the monetary authority follows a standard Taylor rule. Below we provide a sketch of the model and a list of first order and equilibrium conditions.

B.1. Model sketch, CGG model

B.1.1 Households

A representative household has preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\left(\frac{C_t}{A_t}\right)^{1-\tau}}{1-\tau} - \xi_t \frac{N_t^{1+\varphi}}{1+\varphi} \right\}, \tag{40}$$

where utility from consumption is divided by the (growing) level of technology, such as to have a well-defined balanced growth path. The household maximizes the above preferences subject to its budget:

$$P_t C_t + B_t \leq B_{t-1} R_{t-1} + W_t N_t + T_t. \tag{41}$$

B.1.2 Final good firms

Final good firms have production technology

$$Y_t = \left[\int_0^1 Y_t\left(i\right)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{42}$$

where $Y_t\left(i\right)$ are differentiated types of intermediate goods used as production inputs. The final good firm maximizes profits by selling Y_t at P_t and buying $Y_t\left(i\right)$ at prices $P_t\left(i\right)$.

B.1.3 Intermediate goods firms

An intermediate good firm's problem can be split into a (static) cost minimization and a (dynamic) profit maximization problem. The cost minimization problem reads

Table 5: System of model equations, New Keynesian model

(NK1):	$\xi_t N_t^{\varphi} c_t^{\tau} = w_t$
(NK2):	$c_t^{-\tau} = \beta E_t c_{t+1}^{-\tau} \frac{1}{dA_{t+1}} \frac{R_t}{\Pi_{t+1}}$
(NK3):	$p_t^{*\left(1+\frac{\varepsilon\alpha}{1-\alpha}\right)} = (1-\nu) \frac{\varepsilon}{(\varepsilon-1)} \frac{aux_{1,t}}{aux_{2,t}}$
(NK4):	$aux_{1t} = mc_t y_t + E_t \beta\theta \Pi_{t+1}{}^{\frac{\varepsilon}{1-\alpha}} \frac{c_{t+1}^{-\tau}}{c_t^{-\tau}} aux_{1t+1}$
(NK5):	$aux_{2t} = y_t + E_t \beta\theta \Pi_{t+1}^{\varepsilon-1} \frac{c_{t+1}^{-\tau}}{c_t^{-\tau}} aux_{2t+1}$
(NK6):	$S_t y_t = N_t{}^{1-\alpha}$
(NK7):	$S_t^{\frac{1}{1-\alpha}} \equiv (1-\theta)\left(p_t^*\right)^{\frac{-\varepsilon}{1-\alpha}} + \theta\left(\Pi_t\right)^{\frac{\varepsilon}{1-\alpha}} \Delta_{t-1}^{\frac{1}{1-\alpha}}$
(NK8):	$p_t^* = \left[\frac{1-\theta\Pi_t^{\varepsilon-1}}{1-\theta}\right]^{\frac{1}{1-\varepsilon}}$
(NK9):	$mc_t = \frac{1}{1-\alpha} w_t y_t^{\frac{\alpha}{1-\alpha}}$
(NK10):	$c_t = y_t$
(NK11):	$\frac{R_t}{R} = \left(\frac{R_t}{R}\right)^{\rho_R} \left[\left(\frac{\Pi_t}{\Pi}\right)^{\rho_\Pi} \left(\frac{y_t}{y_t^{flex}}\right)^{\rho_y}\right]^{1-\rho_R} e^{\varepsilon_{R,t}}$
(NK12):	$y_t^{flex} = \left[(1-\nu)\frac{\varepsilon}{(\varepsilon-1)}\frac{1}{\xi_t}\right]^{\frac{1-\alpha}{\varphi+\tau(1-\alpha)}}$
(NK13):	$\log\left(dA_t\right) = \rho_A \log\left(dA_{t-1}\right) + (1-\rho_A)\, dA + \varepsilon_{dA,t}$
(NK14):	$\log\left(\xi_t\right) = \rho_\xi \log\left(\xi_{t-1}\right) + (1-\rho_\xi)\, \xi + \varepsilon_{\xi,t}$

$$\min_{N_t(j)} \left\{W_t N_t(j) + MC_t(j)\left[Y_t(j) - A_t N_t(j)^{1-\alpha}\right]\right\},$$

from which an expression for the firm's marginal cost $MC_t(j)$ can be derived. The firm's profit maximization problem, taking as given the demand function the firm faces for its product, is then given by:

$$\max_{P_t(j)} E_t \sum_{k=0}^{\infty} \theta^k \Omega_{t,t+k} \left\{\left[P_t(j) - MC_t(j)\right] Y_t(j)\right\}. \tag{43}$$

B.1.4 System of model equations

Table 5 summarizes the system of equations of the New Keynesian model in terms of stationary allocations and real (relative) prices (i.e., in term of detrended and deflated variables, denoted by lowercase variables), defined as $c_t = \frac{C_t}{A_t}$, $y_t = \frac{Y_t}{A_t}$, $\Pi_t = \frac{P_t}{P_{t-1}}$, $w_t = \frac{W_t}{P_t A_t}$, $b_t = \frac{B_t}{P_t A_t}$, $t_t = \frac{T_t}{P_t A_t}$, $p_t^* = \frac{P_t^*(j)}{P_t}$, $mc_t(j) = \frac{MC_t(j)}{P_t}$, $y_t = \frac{Y_t}{A_t}$, $dA_t = \frac{A_t}{A_{t-1}}$, and where price dispersion is defined as $S_t = \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{\frac{-\varepsilon}{1-\alpha}} dj$. Table 6 summarizes parameter values used in model simulations.

Table 6: Calibration of the CGG New Keynesian model

Symbol	Variable	Value
β	Discount factor	0.99
τ	Coefficient of relative risk aversion	1
ϵ	Elasticity of substitution betw. varieties	9
φ	Inverse Frisch elasticity	5
ρ_π	Coefficient on inflation, Taylor rule	1.5
ϕ_y	Coefficient on output gap, Taylor rule	0
ρ_R	Interest rate smoothing, Taylor rule	0.75
θ	Calvo parameter	0.75
α	1-alfa is the weight on labor in prod. fct.	1/4
ρ_ξ	Autocorrelation, preference shock	0.95
σ_ξ	Volatility, preference shock	0.01
ρ_{dA}	Autocorrelation, TFP growth shock	0.85
σ_{dA}	Volatility, TFP growth shock	0.005

B.2. Results

This section of the appendix lays out results from model simulations for the New Keynesian model. Table 7 and 8 mirror the model versions and results for the RS model in the main text. Table 7 reports model moments for the baseline model with zero trend inflation (NK1), the version with positive trend inflation (NK2) and the version with positive trend inflation and variable capital (NK3). As with the RS model, the trend-inflation augmented model version gives rise to problems of inflated model moments and counterfactual regions over which price dispersion travels, as witnessed in particular by the maximum values of S^{-1} observed over the simulation. As stressed already in the main text, whether or not the NK model is susceptible to counterfactual levels of price dispersion and the resulting problems of unreasonable model moments is ultimately a quantitative question. Simply changing the persistence parameter ρ_{dA} from the reported value in table 6 to 0.5 implies that none of the model versions, also not NK2 or NK3, give rise to any problems and display well-behaved regions for price dispersion, with MAX(S^{-1}) strictly smaller than one and MIN(S^{-1}) not lower than 0.98. Similarly, we never encounter any sign of elevated levels of price dispersion in a model version with trend-stationary shocks.

Table 8 reports model moments for the model versions that feature one of the modeling devices that keep the behavior of price dispersion contained and therefore provide a fix to the problems of inflated moments, paralleling table 2 of the main text. In particular, NK4 considers the case of Rotemberg adjustment costs, NK5 is the model version with a linear-in-labor production function, and NK6 and NK7 are the model versions with inflation indexation, either with respect to steady state inflation or with respect to past

Table 7: Empirical and Model-Based Unconditional Moments

Unconditional Moment	USdata 1961-2007	NK1 $\bar{\pi}=0$	NK2 $\bar{\pi}=2.0\%$	NK3 $Y_t = A_t K_t^{\theta} N_t^{1-\theta}$
SD(C)	0.83	1.59	11.31	6.30
SD(N)	1.71	2.92	4.62	0.97
Mean(π)	3.50	-0.38	-0.41	-0.57
SD(π)	2.52	3.25	6.11	2.23
MEAN(i)	5.72	-0.57	-0.63	-0.87
SD(i)	2.71	3.40	7.99	2.80
MEAN(S^{-1})	0.00	0.98	0.96	1.00
SD(S^{-1})	0.00	0.02	0.13	0.00
MIN(S^{-1})	0.00	0.88	0.21	0.97
MAX(S^{-1})	0.00	1.00	1.35	1.00

Note: Model moments are calculated from the simulated series. ***NK1****: model with labor-only-DRS production function $Y_t = A_t N_t^{1-\alpha}$, zero trend inflation, $\bar{\pi} = 0\%$.* ***NK2****: as in NK1, but: with positive trend inflation $\bar{\pi} = 2\%$.* ***NK3****: as in NK1, but: with positive trend inflation $\bar{\pi} = 2\%$, with variable capital $Y_t = A_t K_t^{\theta} N_t^{1-\theta}$.*

quarter inflation.

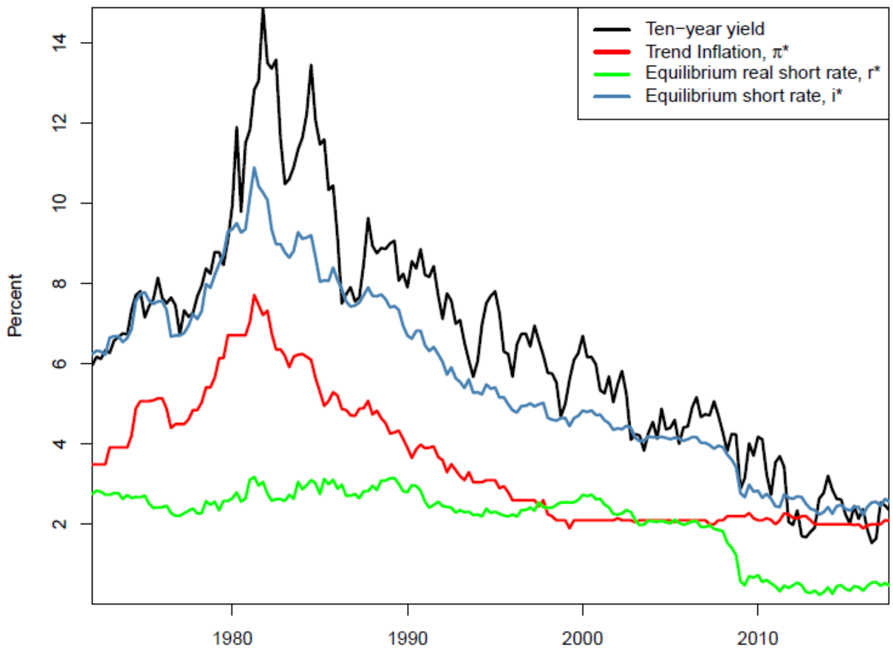
C. Additional figures

Table 8: Empirical and Model-Based Unconditional Moments

Unconditional Moment	NK2 $\pi = 2.0$	NK4 Rotemberg	NK5 $Y_t = A_t N_t$	NK6 $\iota = 0$	NK7 $\iota = 1$
SD(C)	1.59	1.51	1.18	1.59	1.85
SD(N)	2.92	4.42	1.40	2.92	2.82
Mean(π)	-0.38	-0.38	-0.36	-0.38	-0.38
SD(π)	3.25	3.62	4.06	3.25	2.92
MEAN(i)	-0.57	-0.57	-0.54	-0.57	-0.58
SD(i)	3.40	3.40	3.88	3.40	3.27
MEAN(S^{-1})	0.98	1.00	0.99	0.98	1.00
SD(S^{-1})	0.02	0.00	0.01	0.02	0.00
MIN(S^{-1})	0.88	0.98	0.94	0.88	0.98
MAX(S^{-1})	1.00	1.00	1.00	1.00	1.00

Note: Model moments are calculated from the simulated series. ***NK2****: equal to NK2 from Table 7.* ***NK4****: as in NK2, but: with Rotemberg adjustment costs instead of Calvo pricing.* ***NK5****: as in NK2, but: with labor-only-CRS* $Y_t = A_t N_t$. ***NK6****: as in NK2, but: with indexation to steady state inflation.* ***NK7****: as in NK2, but: with indexation to last-period inflation.*

Figure 4: 10Y T-yield,estimates of π^*, the equilibrium real rate, r^*,equilibrium short rate, $i^* = \pi^* + r^*$. The data are quarterly from 1971:Q4 to 2017:Q2. Source: Bauer and Rudebusch (2017)



References

Andreasen, M. M., J. Fernandez-Villaverde, and J. F. Rubio-Ramirez (2018): "The Pruned State-Space System for Non-Linear DSGE Models: Theory and Empirical Applications," *Review of Economic Studies*, 85(1), 1–49.

Andreasen, M. M., and A. Kronborg (2017): "The Extended Perturbation Method: New Insights on the New Keynesian Model," CREATES Research Papers 2017-14, Department of Economics and Business Economics, Aarhus University.

Ascari, G. (2004): "Staggered Prices and Trend Inflation: Some Nuisances," *Review of Economic Dynamics*, 7(3), 642–667.

Ascari, G., E. Castelnuovo, and L. Rossi (2011): "Calvo vs. Rotemberg in a trend inflation world: An empirical investigation," *Journal of Economic Dynamics and Control*, 35(11), 1852–1867.

Ascari, G., and T. Ropele (2009): "Trend Inflation, Taylor Principle, and Indeterminacy," *Journal of Money, Credit and Banking*, 41(8), 1557–1584.

Ascari, G., and L. Rossi (2012): "Trend Inflation and Firms Price-Setting: Rotemberg Versus Calvo," *Economic Journal*, 122(563), 1115–1141.

Ascari, G., and A. M. Sbordone (2014): "The Macroeconomics of Trend Inflation," *Journal of Economic Literature*, 52(3), 679–739.

Backus, D., A. W. Gregory, and S. E. Zin (1989): "Risk Premiums in the Term Structure," *Journal of Monetary Economics*.

Bauer, M. D., and G. D. Rudebusch (2017): "Interest Rates Under Falling Stars," Discussion paper.

Boneva, L. M., R. A. Braun, and Y. Waki (2016): "Some unpleasant properties of loglinearized solutions when the nominal rate is zero," *Journal of Monetary Economics*, 84(C), 216–232.

Cieslak, A., and P. Povala (2015): "Expected Returns in Treasury Bonds," *Review of Financial Studies*, 28(10), 2859–2901.

Clarida, R., J. Gali, and M. Gertler (1999): "The Science of Monetary Policy: A New Keynesian Perspective," Discussion Paper 4.

Cogley, T., G. E. Primiceri, and T. J. Sargent (2010): "Inflation-Gap Persistence in the US," *American Economic Journal: Macroeconomics*, 2(1), 43–69.

Constancio, V. (2015): "The role of stress testing in supervision and macroprudential policy," Keynote address at the london school of economics conference on stress testing and macroprudential regulation: a trans-atlantic assessment.

Costain, J., and A. Nakov (2011): "Distributional Dynamics under Smoothly State-Dependent Pricing," *Journal of Monetary Economics*, 58, 646–665.

Den-Haan, W. (1995): "The term structure of interest rates in real and monetary economies," *Journal of Economic Dynamics and Control*.

Golosov, M., and R. E. Lucas (2007): "Menu Costs and Phillips Curves," *Journal of Political Economy*, 115, 171–199.

Kliem, M., and A. Meyer-Gohde (2017): "(Un)expected Monetary Policy Shocks and Term Premia," Bundesbank Discussion Paper No. 30/2017. 2017-015, Berlin.

Miao, J., and P. V. Ngo (2019): "Does Calvo Meet Rotemberg at the Zero Lower Bound?," *Macroeconomic Dynamics, forthcoming*.

Midrigan, V. (2011): "Menu Costs, Multiproduct Firms, and Aggregate Fluctuations," *Econometrica*, 79, 1139–1180.

Nakamura, E., J. Steinsson, P. Sun, and D. Villar (2016): "The Elusive Costs of Inflation: Price Dispersion during the U.S. Great Inflation," NBER Working Papers 22505, National Bureau of Economic Research, Inc.

Rudebusch, G. D., and E. T. Swanson (2012): "The Bond Premium in a DSGE Model with Long-Run Real and Nominal Risks," *American Economic Journal: Macroeconomics*, 4(1), 105–43.

Stock, J. H., and M. W. Watson (2007): "Why Has U.S. Inflation Become Harder to Forecast?," *Journal of Money, Credit and Banking*, 39(s1), 3–33.