YIELD CURVE DYNAMICS AND FISCAL POLICY SHOCKS

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Abstract
We use an affine term structure model with time-varying macro trends and a vector autoregression model to investigate the response of the US Treasury yield curve to changes in fiscal policy. By accounting for the timing of the fiscal policy in the shock identification we can separate the effect of news about future increases in government spending from the effect of innovations in changes of current government expenditures. Further, we use the Baker, Bloom, and Davis (2016) uncertainty index dataset to explain the flight to quality type of events. By controlling for the low frequency movement in yields and the decomposition of yield to risk neutral rates and term premia we show that the news channel is driven by a cautious response of agents to an increase in projected future government spending and leads to a drop in yields. This result contrasts with shock into contemporaneous spending which has no significant impact on bond yields.

JEL classification: C12; C22; C52.
Key words: Government Expenditures, Affine Term Structure Model, Time-varying Macro Trends.

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NON-TECHNICAL SUMMARY

The term structure of interest rates is the key source of information in macroeconomics and finance. The yield curve has been established as an essential tool in predicting the business cycle; it is a fundamental input in asset pricing and debt management and plays crucial role in financial stability assessment. The yield curve serves as a core input for the macroprudential policy to test the financial stability of banking and insurance sector. The health of the financial institutions is assessed by testing their balance sheet sensitivity with respect to movements in the yield curve implied by a set of scenarios of possible future economic outcomes.

It has been long known that government policy through its monetary policy conduct is the key element determining the level and shape of the yield curve. This is because the short rate is essentially a regulated price by the monetary policy authority and long rates are nothing else than risk adjusted expectations about future short rates.

In this paper we argue that not just a monetary policy but also the fiscal policy plays important role in shaping the yield curve. Hence, fiscal policy impact on yield curve constitutes yet another, important, channel through which fiscal policy impacts the real economy and asset prices.

More specifically, our work contributes to the no-arbitrage dynamic term structure literature by introducing government spending variable and showing that it is an important driver of bond prices across maturities. The importance of fiscal variables has been documented in this literature by Dai and Philippon (2005) who point to the fact that government deficits can temporarily affect long term interest rates. The impact of changes in government expenditures received however much less attention and it has been believed that government spending has marginal impact on the yield curve. This is a surprising result as the textbook economic theory predicts that an exogenous increase in public spending should lead to a rise in aggregate demand driving interest rates up. In addition, if the rise in expenditures is financed by debt the bond supply literature documents the positive relationship between supply of outstanding government bonds and interest rates.

We build upon the fiscal foresight literature which argues that economy responds primarily to shocks to agents’ expectations about fundamentals and future policy. The relevance of this type of shocks (news shocks) for the yield curve has been demonstrated by Kurmann and Otrok (2013). They show that the news about future total factor productivity can explain more than 50 of the unpredictable movements in the slope of the yield curve.

We show that the shock identification that accounts for anticipation results in precautionary responses of economic agents to the changes in fiscal policy. This means that bonds provide an insurance vehicle against future risks and hence yields decrease in response to news about the change in government expenditures. On the other hand, the response of yields to positive
surprise spending shock increases yields. However, the response of yields is weak and with low statistical significance. We also confirm the result of Gale and Orszag (2003) and others that contemporaneous shocks into debt to GDP ratio increase the yields across the maturity profile.

Further, we show that fiscal and geopolitical uncertainty can account for a large part of variance at the long end of the yield curve. Bauer (2017) attributes the flattening to the drop in expected future returns due to changes in inflation premium. Our result highlights yet another channel, we argue similarly to Duffee (2018) that uncertainty about the future short real rates plays an important role. To study the role of uncertainty we replace the news shock by a measure of fiscal policy uncertainty. The response of yields to the rise in uncertainty remains identical for the short yields, but not for the long yields. This finding suggests that treasuries are no longer viewed as a safe haven asset if the rise in uncertainty is generated by U.S. fiscal policy. This result contrasts with our other finding showing that increase in the global uncertainty (for which treasuries do serve as a safe haven), triggers a drop in long yields.
1. **INTRODUCTION**

What is the impact of government spending on the term structure of interest rates? The existing literature provides little guidance as it is either predominantly focused on studying implications of fiscal policy for the real economy or the reverse - impact of changes in bond prices on the real economy. Empirical research studying directly the link between fiscal policy and the yield relies mainly on the simple least-squares estimates reduced to single bond maturity (see Evans and Marshall (2007) or Laubach (2009) Gale and Orszag (2003)). We bridge these literature streams and study the effect of government spending on the yield curve in a unified framework allowing for the bi-directional relationship between the real economy and the whole maturity spectrum of bond prices. We estimate the affine term structure macro-finance model where we account for the small sample biases as in Bauer and Rudebusch (2017) and for the role of timing in the impact of fiscal policy on the dynamics of U.S Treasury yields curve by following the news literature (Ramey (2011), Kurmann and Otrok (2013)). We show that fiscal policy can explain much of the dynamics in term structure of interest rates once we properly account for shock timing.

Our work contributes to the no-arbitrage dynamic term structure literature by introducing government spending variable and showing that it is an important driver of bond prices across maturities. Dai and Philippon (2005) points to the fact that government deficits can temporarily affect long term interest rates. The impact of changes in government expenditures received however much less attention and it has been believed that government spending has marginal impact on the yield curve (see for instance, Evans and Marshall (2007). This is a surprising result as the textbook economic theory predicts that an exogenous increase in public spending should lead to a rise in aggregate demand (i.e. Baxter and King (1993) driving interest rates up (i.e. Fisher and Turnovsky (1992)). In addition, if the rise in expenditures is financed by debt the bond supply literature documents the positive relationship between supply of outstanding government bonds and interest rates (see Krishnamurthy and Vissing-Jorgensen (2007) for literature review).

Ramey (2011) is among the first to forcefully document in empirical study the importance of fiscal foresight in the response of the economy to rises in public expenditures. The fiscal

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3Impact of fiscal policy for the real economy has been extensively covered in the literature on fiscal multipliers (see e.g. Christiano, Eichenbaum, and Rebelo [2011] for a survey). Research studying impact of bond prices for real economy is dominated by financial frictions literature see e.g. Brunnermeier, Eisenbach, and Sannikov [2013] for a survey)

4Gale and Orsag (2003) provide extensive literature review on how the timing of fiscal policy in case of deficit and debt matters for the response of the yields. For instance, Barth (1991) surveys 42 studies and finds: from 19 studies with projected deficits 13 have positive, 5 mixed effects, 1 no effect. Gale and Orsag (2003) redo Barth (1991) and find: 18 studies have positive effect, 6 mixed effects, 19 not significant or negative. Similar conclusion found by Mankiw (1999). Often cited papers by Evans (1987) or Plosser (1982) find no effect. Ardagna, Caselli, and Lane (2007) use both a simple static estimation and a vector autoregression model for a panel of countries and show that an increase in the primary government deficit increases the long-term yields. However, in the case of an increase of the government debt, the yields are affected only for the above-averagely indebted countries. Laubach
foresight literature (see Leeper, Richter, and Walker (2012)) complements the ‘news’ literature (Beaudry and Portier (2006) and Barsky and Sims (2011)) which posits that business cycles arise on the basis of expectations of future fundamentals rather than on the impact of shock. The importance of news shocks for yield curve has been established in Kurmann and Otrok (2013); they show that it is the news about future total factor productivity which explains more than 50% of the unpredictable movements in the slope of the yield curve. The effect of news about government spending on the yield curve has not however been studied in the literature. Yet intuitively many fiscal policy measures are known well in advance. The lags in decision and implementation can be demonstrated by many examples. Trump’s fiscal package to boost infrastructure spending has been debated since he won the election. Obamacare 5 was discussed for more than a year before coming into force and the implementation was only gradual. Ramey (2011) lists other examples related to defense spending as the aftermath of 9/11 or Soviet invasion of Afghanistan, where the rise in defense spending was anticipated in advance. Leeper, Richter, and Walker (2012) documents that non-negligible portion expenditures are only foreseen imperfectly. Building on the fiscal foresight literature, we distinguish between the:

i) news shock, which is the shocks to agents’ expectations about future policy, ii) the surprise shock, which results from the revision in expectations at the time the expenditures are realized, and the iii) uncertainty shock which reflects the changes in the level of insecurity in forming the expectations about future public expenditures. Accounting for timing allows us to better disentangle the channels of transmission between fiscal policy shocks and yields.

We show that the shock identification that accounts for anticipation results in precautionary responses of economic agents to the changes in fiscal policy. This means that bonds provide an insurance vehicle against future risks and hence yields decrease in response to news about the change in government expenditures. On the other hand, the response of yields to positive surprise spending shock increases yields. However, the response of yields is weak and with low statistical significance. We also confirm the result of Gale and Orszag (2003) and others that contemporaneous shocks into debt to GDP ratio increases the yields across the maturity profile.

Further, we show that fiscal and geopolitical uncertainty can account for a large part of variance at the long-end of the yield curve. Bauer (2017) attributes the flattening to the drop in expected future returns due to changes in inflation premium. Our result highlights yet another channel, we argue similarly to Duffee (2018) that uncertainty about future short real rates plays an important role. To study the role of uncertainty we replace the news shock by the measure of fiscal policy uncertainty. The response of yields to the rise in uncertainty remains identical for the short yields, but not for the long yields. This finding suggests that Treasuries are no longer viewed as a safe haven asset if the rise in uncertainty is generated by U.S. fiscal policy. This (2009) shows the upward effect of fiscal expansion on the long-term yields by comparing the budget deficit forecasts with the long-horizon forward rates.

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result contrasts with our other finding showing that increase in the global uncertainty (for which Treasuries do serve as a safe haven), triggers a drop in long yields.

The important novelty of our approach is that we follow Bauer and Rudebusch (2017) and attribute the transmission channels to either a nominal component of the yields (expected inflation), real rate component or the term premia. The model by Bauer and Rudebusch (2017) allows for persistent trends in the yields, which allows us to avoid the small sample bias issue when decomposing the yield curve into the components (see Bauer, Rudebusch, and Wu (2012)). Additionally, we check robustness of our results with respect to the lower bound environment. We reevaluate the responses using the shadow-rate affine macro-finance model building on the methodology from Krippner (2013) and Christensen and Rudebusch (2014). This framework incorporates the zero lower bound (ZLB) constraint and thus allows us to extend the sample period of the aftermath of financial crisis.

The rest of the paper is organized as follows. The second section introduces the term structure model and the methodology to estimate the linkages between the fiscal policy and the yield curve. Section 3 shows the results of estimated term structure models, extracted factors and yield components. In section 4, we describe the data used in the vector autoregression analysis and discuss the model identification. In section 5, we evaluate and discuss the responses of yields to the fiscal policy impulses within our baseline model. Section 6 offers additional views on mechanisms of the transition of the shocks. Section 7 shows robustness checks, whereas section 8 checks the validity of the results under the lower bound. Finally, section 9 concludes. An appendix offers detailed descriptions of the term structure model, lag selection and the robustness checks.

2. Empirical Approach

To analyze the effect of the fiscal policy shocks on the yield curve, we adopt the following strategy. The first step involves yield modeling. The yield curve comprises multiple maturities. To reduce its dimensionality, we follow an approach commonly used in the literature since the pivotal work of Ang and Piazzesi (2003): we use an affine term structure model that imposes structural relations among yields of various maturities. This allows us to model the whole yield curve using only several factors. The no-arbitrage conditions imbedded in the model specification allow us to obtain yield components: the risk-neutral yield and the term premium. Both components may react differently to the macroeconomic developments, which has been observed historically, for example, in connection with so-called Greenspan’s conundrum. The Fed exercised a policy of its key interest rate at or below 0.5% from December 2008 to December 2016. A discussion of monetary policy at the lower bound can be found in Buitie (2009) or Swanson and Williams (2014).

The conundrum was related to the inability of the U.S. Federal Reserve Board to influence the longer yields, despite significant adjustments of the Fed funds rate over the 2005–2007 period (FRB, 2017). Backus and Wright (2007) later explained that this result was caused by the decreasing term premium, which offset the increase of the risk-neutral yields.
ability of the yield model to decompose the yields (and the estimated yield responses) is therefore crucial for a qualified assessment of the effects of shocks on the yield curve.

We use the methodology of Bauer and Rudebusch (2017) to find the yield factors. The factors thus represent cyclical and trend components of both inflation and real rate plus a latent variable governing the price of risk. Such representation is plausible, since it provides direct motivation to link these macro-financial factors with a set of other variables, including measures for fiscal policy. Therefore, in the second step, we evaluate the responses of the yield factors (and therefore the yields and the yield components) to a set of macroeconomic and fiscal policy shocks using a vector autoregression (VAR) model. Calculated impulse response functions and decomposition of forecast error and historical variances are used as the main tool to infer the linkages.

2.1 Term Structure Model

To characterize the yield curve by several factors, we use the affine term-structure model by Bauer and Rudebusch (2017). This model imposes multiple restrictions on both the transition of shocks under the data-generating measure and the specification of the price of risk. Most importantly, it assumes that the yield factors include an equilibrium inflation \( \pi^* \) and an equilibrium real rate \( r^* \), which follow a random walk process. Such assumption is crucial since it allows the abandonment of the mean-reverting nature of the yield factor processes implicitly imposed by the canonical term structure model, which leads to imprecise identification of the yield components.

The model by Bauer and Rudebusch (2017) results in a representation of yields \( y_t(\tau) \) with maturity \( \tau \) as an affine function of the five factors plus the convexity component \( \text{conv}(\tau) \):

\[
y_t(\tau) = \pi_t^* + b_{\pi}(\tau) \pi_t^c + b_r(\tau) r_t^c + b_f(\tau) f_t + \text{conv}(\tau)
\]

where \( b_{\pi}, b_r \) and \( b_f \) are the factor loadings related to the inflation cycle \( \pi_t^c \), the real rate cycle \( r_t^c \) and the term premium-governing latent factor \( f_t \). The factor loadings and the convexity component are dependent on maturity. Detailed derivation, which follows directly from Bauer and Rudebusch (2017), is shown in Appendix I.

The estimated value of the factor loadings for various maturities is shown in Figure 1. It illustrates the core implications of the model. The term premium is equal to zero for the short rate, which is therefore defined as a sum of the inflation and real rate trends and cycles. On the contrary, the long yields are relatively less affected by the cyclical components and their dynamics are therefore mainly driven by the inflation and real rate trends and the term premia. Such setup allows for joint modeling of the long-yield trends in yields represented by the trend factors, but at the same time it allows for the situation where long and short yields decouple due to divergent evolution of macroeconomic cycles and the term premia.
To estimate the model, we use a state-space representation of the yield factors dynamics, which is completed by defining the transition equation as first-order vector autoregression process:

$$X_t = \Phi X_{t-1} + \Sigma \epsilon_t$$  \hspace{1cm} (2)

where $X_t = [\pi^*_t, \pi^c_t, r^*_t, r^c_t, f_t]'$ is the vector of the yield factors, $\Phi = diag(1, \phi_{\pi}, 1, \phi_{r}, 1)'$ is the VAR(1) transition matrix with non-unit diagonal elements assumed to be less than one in the absolute value, $\epsilon_t = [\epsilon_{t,\pi}, \epsilon_{t,\pi^c}, \epsilon_{t,r}, \epsilon_{t,r^c}, \epsilon_{t,f}]'$ $\sim$ $N(0, I_5)$ are iid random disturbances and $\Sigma$ is a diagonal matrix of standard deviations of the random disturbances.

We employ maximum likelihood with Kalman filter for filtering the factors. To do so, however, it is necessary to impose several restrictions so that the model is identified. Following Bauer and Rudebusch (2017), we consider the trend variables $\pi^*$ and $r^*$ as observed. We use the estimations of $r^*$ from Laubach and Williams (2003) and a measure for perceived inflation target rate of FED for $\pi^*$.

Additionally, to help the model to distinguish between the inflation cycle and the real rate cycle, we enrich the model by a proxy for the real short rate, which we obtain as a difference of the Fed Funds Rate from FED (2018a) and the short-term inflation expectations from FRB (2018). Other factors are inferred from the model.

### 2.2 Measuring the Impact of Shocks

The presented model falls into the category of macro-finance models, as it links the yields to evolution of macroeconomic factors. However, for the purposes of our analysis, fiscal policy as a source of yield variation needs to be introduced as well.

The first option would be to extend the set of yield factor of the affine model by the additional

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*See discussion in Bauer and Rudebusch (2017) on source and nature of the perceived inflation target rate.*
variables directly. Such an approach has been used in the literature since the pivotal work of [Ang and Piazzesi (2003)]. However, introducing the fiscal factors into the model by [Bauer and Rudebusch (2017)], already rich in the assumptions on its structure, would require imposing additional assumptions on the fiscal factors in relation to the other macro-variables and simultaneously could result in a worsened convergence of the model.

Therefore, we instead use a two-step approach. First, we estimate the [Bauer and Rudebusch (2017)] model per se and then use the obtained yield factor as one of the inputs into a vector autoregression model (VAR). As shown by [De Pooter, Ravazzolo, and Van Dijk (2010)] and further supported by [Joslin, Singleton, and Zhu (2011)], the results of two-step and one-step approach are usually not significantly different, while the two-step approach is generally more robust.

The two-step approach also allows us to exclude the inflation trend and equilibrium real rate from the VAR analysis. By excluding the trend components of yields we assume that the fiscal policy affects the yields only on the business cycle frequency. We thus follow much of the literature on public capital where government expenditures are seen as wasteful expenditures. The omission of the trend variables significantly improves robustness of the results, since the clearly non-stationary trends in yields are excluded from the analysis.

We calculate the responses of the yield factors to the impulses of observable variables from the VAR model. Next, using the interest rate model loadings from Equation 1, we obtain the responses of the yields. The nature of the yield factors in the term structure model allows us to directly interpret, whether the impact of the the macro-financial shocks to yields is transmitted through the risk-neutral components or the term premium.

We proceed with the identification of the model. In the first step of the two-step approach, the variance in yields is attributed to the yield factor only. The variance unexplained by inflation and real rate trend and cycle is attributed to the variance of the latent factor. Therefore, in the VAR analysis utilizing Choleski decomposition of the shocks, it is meaningful to order the latent factor at the end of the set of VAR variables. That ensures that the shocks from the fiscal and control variables in the VAR model are correctly attributed to the changes in the latent factors [Kilian and Lütkepohl, 2017]. The macroeconomic factors are included in the beginning of the set of variables.

The Choleski decomposition could appear too simple to identify the fiscal policy shocks as long as

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9 For instance, Barro (1990), Baxter and King (1993), Gramlich (1994) provides literature survey. We are aware of the literature on productive government spending, for instance Turnovsky (1997) or Agenor (2013). This type of expenditure consists however of less than 15% of total government expenditures and there is no evidence in the literature that it affects inflation trend and real interest rate.

10 In case the trend variables were included into the VAR model, all the eigenvalues of the companion matrix are smaller than 1 in modulus. However, the largest eigenvalue in modulus exceeds 0.99. Therefore, the responses of the variables to shocks are nearly persistent, which contradicts the cycle-smoothing nature of monetary and fiscal policies.
as these would be represented by the historical series of the government spending. Therefore, we build on the narrative approach, in the spirit of [Ramey (2011)], to identify the
government spending shocks (see section 5 for details). In our case, the narrative approach
centers on identifying the shocks in the fiscal policy projections rather than the realized bud-
getary series, which allows us to identify the shocks at the moment the future changes in the
fiscal policy become expected by the economic agents. In the light of this approach, we con-
sider the Choleski decomposition as sufficient, since the shocks are identified by the selected
time series rather than the model restrictions. We also support the robustness of our results
by robustness checks. We use bootstrapping to construct confidence intervals for our results.
More specifically, to construct the confidence intervals, we employ the Bonferroni band ad-
justed to avoid excessive conservatism, following Lütkepohl, Staszewska-Bystrova, and Winker

3. YIELD FACTORS AND COMPONENTS: EMPIRICAL RESULTS

In this section, we illustrate the results of the interest rate model applied to the recent U.S. data.
We use zero-coupon Treasury yields from Gurkaynak and Wright (2007). We include maturities
1–15 years in our sample, which we further extend by 3-month and 6-month Treasury bill yields
from FED (2018a). Since we use macroeconomic variables observed at a monthly frequency in
section 5 we gather the end-of-month yields. The sample period ranges from November 1985 to August 2018. The beginning of the sample is constrained by the availability of the data (especially fiscal projections data and uncertainty indices – see section 5). The start of
the sample also ensures the period of very high monetary policy rate from the first half of the
80's is not included in the sample, and thus we may assume a relative homogeneity of the U.S.
monetary policy conduct over the whole sample.

We present the evolution of U.S. government bond yields over the selected period in Figure 2.
In this period, the yield curve was mostly upward-sloping, with few exceptions prior to the 1990,
2001 and 2008 crises. Since the end of 2008, the lower bound proximity has apparently been
effective, as the short end of the yield curve fluctuated around the zero level with a limited
volatility. At the end of 2015, the lift-off of the short yields began to take place. However, the
long end of the yield curve gradually decreased over the whole period. Since the model of
Bauer and Rudebusch (2017) includes factors that follow random walk, the zero lower bound
has lower impact on the validity of the term structure model than in the case of the Gaussian
stationary models (see Krippner, 2015 for discussion). However, still, to support the validity

11 For example, Dai and Philippon (2005) instead use the Blanchard and Perotti (2002) approach to identify the
fiscal policy shocks. However, as Ramey (2011) shows, the identification is still not able to identify the shocks correctly and a narrative approach is needed instead.
of our results, we re-estimate our results using a shadow-rate model by Krippner (2013) in section 8 to demonstrate that the conclusions made in the text below are robust to the lower bound proximity.

Figure 2: Yield Data (1985/11–2018/08)

Note: The shaded areas show the NBER-defined crises.

The obtained yield factors are shown in Figure 3. The perceived inflation target rate representing \( \pi^* \) and the \( r^* \) are gradually decreasing over the sample period, which well explains the long-term decreasing trend in yields. The inflation cycle and real rate cycle are responsible for the fluctuation of the short yields around the trend. The latent factor is, equivalently, the source of variability in long yields around the long-term trends. Most importantly, the latent factor \( f \), which governs the term premia, is countercyclical.

Given the factor and estimated parameters, the yields can be decomposed into the components (Figure 4). We calculate the inflation component for each maturity \( \tau \) as a sum of the trend and cyclical component \( \pi^*_t + b_\pi(\tau)\pi^*_t \). Equivalently, the real rate component is given as \( r^*_t + b_r(\tau)r^*_t \). Finally, the residual component is formed by the term premium and the time-invariant convexity (which is constant over the time) \( b_f(\tau)f_t + conv(\tau) \). For the short maturity, the inflation and real rate components dominate the yield. In contrast, in the case of yields of bonds with a longer maturity, the importance of the term premium proportionally increases.

Further, we compare our term structure factor estimates with related studies. Our term premium is counter-cyclical and growing in crises periods (shaded areas). The counter-cyclical is an important argument raised in the discussion of Bauer, Rudebusch, and Wu (2014). The behavior of term premia is in line with similar studies which control for small sample bias and use the affine term structure models (as in Christensen and Rudebusch (2016) and Bauer and Rudebusch (2017)). In the Figure 4 (bottom panels), we compare our estimates with the Adrian, Crump, and Moench (2013) term premia (henceforth ACM), which are publicly available at FED...
The cyclicality for both our and ACM term premia are similar. ACM term premia, on the other hand, follow a downward trend, whereas our term premia are oscillating within a stable range. This is the consequence of the random-walk variables $\pi^*$ and $r^*$ in Bauer and Rudebusch (2017) that absorb the trend in yields.

4. VAR Model Variables and Specification

The core part of the analysis is linking the dynamics of the yield factors to fiscal variables. To do so, we use a set of fiscal variables accompanied by control variables which jointly enter the VAR model.

Fiscal policy impacts yields in response to: i) news about future policy, ii) revisions in expectations at the time the expenditures are realized, iii) changes in the uncertainty surrounding the future expenditures. News shocks reflect the fact that economic agents adjust their behavior when the fiscal policy step is introduced (becomes anticipated). For example, an anticipated fiscal stimulus may improve agents expectation about future economic situations and therefore drive yields upwards. This situation was observed after the 2016 presidential election (Bauer, 2017). However, this channel can be viewed in terms of Ricardian equivalence (Barro, 1974), which suggests that economic agents will increase savings after an expansionary fiscal policy shock, which will drive the yields downward. Ramey (2011) shows that the response depends on the shock identification. As Ramey (2011) shows, standard VAR identification implies a growth of consumption after a positive government spending shock. In contrast, if the adjust-
Figure 4: Yield Components (in %, 10/1993–12/2017)

Note: The shaded areas show the NBER-defined crises. ACM TP denote term premia estimated from Adrian, Crump, and Moench (2013).

ments in government spending are allowed to be anticipated several quarters ahead, their effect is opposite, i.e. consumption lowers. Therefore, in our analysis, we similarly aim at properly identifying the fiscal policy shocks.

We read the impulse response function from the estimation as follows: The news shocks affect the government bond demand as a consequence of the shifts in agents’ decisions regarding both their portfolio allocation and their economic behavior. The government spending, on the other hand, affects also the government bond market through the governmental financing. As long as the government needs to finance the additional spending, for instance, it increases the bond supply, which can be expected to drive the yields upwards, ceteris paribus. The increased need for financing may also increase uncertainty of economic agents regarding longer-term fiscal policy, which further affects the bond demand. We measure these effects by including a government debt variable in the model.

We also document the effect of fiscal policy uncertainty. We replace the anticipated fiscal policy shocks by a proxy for fiscal policy uncertainty. As we show in section 6 both fiscal policy spending shocks and fiscal uncertainty shocks have a similar effect, which further promotes the importance of the saving motives within the agents’ responses to the fiscal policy shocks.

Table 1 includes the set of variables, which we use in our baseline model. When identifying the fiscal policy shocks, we aim at identifying the government spending shocks before they appear in the government balances. Ramey (2011) uses a news-based narrative approach to identify the shocks. We stick to the forward-looking approach but calculate the shocks differently in order to obtain a larger number of shocks. To obtain monthly data, we use fiscal projections

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We infer the shocks from observed changes in the projections up to 5 years ahead. We define these changes in real terms and further discount them to the moment when the change was published – for discounting, we use the government nominal yields in the particular moment adjusted to real terms by the projected inflation, which is part of the [CBO (2018)](https://www.cbo.gov) data. Finally, we express the shocks as a relative monthly change in these projected outlays. The obtained government spending shocks are shown in [Figure 5](#). As the figure shows, the largest shock is observed in 2009, when the consequences of the financial crisis caused significant fiscal policy costs. In the baseline estimation, we identify the shocks as changes in projected government outlays; in sensitivity testing, we evaluate the effects of using a monthly difference in relative budget balance instead.

The fiscal projections from the Congressional Budget Office were similarly used by [Laubach and Williams (2003)](https://www.nber.org/papers/w10829), who focused on measuring the relationship between the change in projected deficit and long-horizon forecasts of government yields (unlike our focus on the relationship between the outlays and the contemporary adjustment in yields). Related approaches using a forward-looking view on the government spending shocks also include already mentioned news-based evidence [Ramey (2011)](https://www.nber.org/papers/w17477) and the usage of the Survey of Professional Forecasters by [Leeper, Richter, and Walker (2012)](https://www.nber.org/papers/w20168).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Category</th>
<th>Description</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Industrial Production Index</td>
<td>Macro</td>
<td>annual log-difference</td>
<td>FRED (2018)</td>
</tr>
<tr>
<td>CPI Cycle</td>
<td>Yield+Macro</td>
<td>estimated</td>
<td>Term structure model</td>
</tr>
<tr>
<td>Real Rate Cycle</td>
<td>Yield+Macro</td>
<td>estimated</td>
<td>Term structure model</td>
</tr>
<tr>
<td>Economic Policy Uncertainty</td>
<td>Control</td>
<td>annual difference</td>
<td>Baker, Bloom, and Davis (2016)</td>
</tr>
<tr>
<td>Government Spending</td>
<td>Fiscal</td>
<td>changes in fiscal projections</td>
<td>CBO (2018)</td>
</tr>
<tr>
<td>Latent Factor</td>
<td>Yield</td>
<td>estimated</td>
<td>Term structure model</td>
</tr>
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</table>

The effects of government financing on the yields through bond supply are measured using the outstanding amount of government debt from [Treasury (2018)](https://www.treasury.gov). We measure the debt in relative terms, related to GDP. The final series are specified as year-over-year difference in the ratio. As [Figure 5](#) shows, the largest increase in the ratio appeared over 2009-10, slightly lagged compared to the peak in the forward-looking government spending variable.

It must be noted that unlike [Ramey (2011)](https://www.nber.org/papers/w17477), we use all U.S. budget income and outlay categories, i.e., we do not restrict the sample to defense spending only. To ensure that the fiscal policy shocks are correctly identified (orthogonal to business cycle), we use control variables. The a monthly frequency, the number of narrative-based defense spending shocks would be too low to allow for robust results.

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**Table 1: Baseline Model Variables**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Category</th>
<th>Description</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Industrial Production Index</td>
<td>Macro</td>
<td>annual log-difference</td>
<td>FRED (2018)</td>
</tr>
<tr>
<td>CPI Cycle</td>
<td>Yield+Macro</td>
<td>estimated</td>
<td>Term structure model</td>
</tr>
<tr>
<td>Real Rate Cycle</td>
<td>Yield+Macro</td>
<td>estimated</td>
<td>Term structure model</td>
</tr>
<tr>
<td>Economic Policy Uncertainty</td>
<td>Control</td>
<td>annual difference</td>
<td>Baker, Bloom, and Davis (2016)</td>
</tr>
<tr>
<td>Government Spending</td>
<td>Fiscal</td>
<td>changes in fiscal projections</td>
<td>CBO (2018)</td>
</tr>
<tr>
<td>Latent Factor</td>
<td>Yield</td>
<td>estimated</td>
<td>Term structure model</td>
</tr>
</tbody>
</table>

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**YIELD CURVE DYNAMICS AND FISCAL POLICY SHOCKS**

Working Paper NBS

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selection of control variables is based on the related macro-financial literature. De Pooter, Ravazzolo, and Van Dijk (2010) summarizes the common approach to include a real-activity variable, a price-dynamics variable and a monetary policy variable. From these we first employ the industrial production index (IPI) as a proxy for the real activity. We prefer IPI to GDP because of its monthly frequency. At the same time, the correlation of IPI and GDP growth rates is high, which makes IPI a good proxy for the real activity. We use an annual log-difference in the IPI index and order it first in the VAR vector: using the Choleski identification, we assume that the transition of the other shocks to the real economy is lagged at least one month. The price dynamics and monetary policy are already represented in the model by the inflation cycle and real rate cycle factors from the term structure model. Given their sluggishness in response to financial and fiscal impulses, they are ordered after the IPI.

We also consider a variable representing the general uncertainty perceived by the financial markets. The variable is ordered beyond the macro variables, which allows us to interpret the innovations to this variable as either macro-unrelated (i.e., caused by non-economic events only) or as an over-reaction to the macroeconomic news. The motivation for the inclusion of shocks to uncertainty into the model can be found in recent events that triggered a flight to quality behavior, inducing a growth in demand for U.S. bonds. An example of such an event is the Brexit referendum in the UK in June 2016, which led the U.S. yields to hit record lows in July 2016, although the U.S. economic situation was generally improving in that period. As a proxy for the general uncertainty, we use the Global Economic Policy Uncertainty Index (GEPU) as calculated by Baker, Bloom, and Davis (2016). Using this proxy ensures that the innovations to variables representing the fiscal policy do not include any additional source of uncertainty with which they are correlated, since they are controlled by GEPU. The GEPU calculation is news-based and is obtained by calculating the number of appearances of key words in the news (see Baker, Bloom, and Davis (2016). To test the robustness of our results, we also consider the VIX index (CBOE 2018) as an alternative uncertainty proxy.

The two fiscal variables are ordered after the control variables. Finally, the yield latent factor is ordered at the end of the VAR vector. Since the latent factor comprises of all information affecting the term premia of yields, its inclusion at the end results in an interpretation of innovations to this factor as the residual shocks unexplained by any other variable in the model.

We test the stationarity of this sample using an Augmented Dickey-Fuller test. At the 5% significance level, we are able to reject the null hypothesis of the unit root for all variables except the government debt change, for which the null hypothesis is, however, rejected at the 1% level.

The set of final variables entering the baseline VAR model is shown in Figure 5.
Note: The units of the y-axes are not displayed since the variables are standardized to a zero mean and one-unit standard deviation. The year marks denote the beginning of each year. The government spending variable represents change in spending projections over horizon up to 5 years.

5. Results

Using seven variables, a VAR(5) model is estimated. We consider the five lags as the optimal specification based on a combination of model completeness and a parsimony that also reflects the results for the information criteria (see Appendix II).

Using the estimated VAR model, the responses of yield factors to shocks to fiscal and control variables are calculated first. Afterwards, using the estimated Equation 1 parameters, the responses of yield factors are then transformed to the responses of yields. Results for yields of selected maturities over the four year response horizon are shown in Figure 6, results for the whole yield curve at selected response horizons are shown in Figure 7. The figures display the estimated response together with 68% confidence bands obtained by bootstrapping.\(^{13}\)

The responses of yields to shocks in IPI and the uncertainty follow an economic intuition. A positive shock to real activity temporarily pushes the short end of the yield curve upwards while keeping the long yields mostly unchanged, the yield curve therefore rotates. The response is

\(^{13}\)Using the asymptotic normality, the 68% level roughly represents single standard deviation confidence band, which is common in the literature to measure the effects of the fiscal policy – see, for example, [Ramey 2011].
transmitted mostly through the real rate cycle factor, which reflects the monetary policy tighten-
ing after the positive real activity shock. After an upward uncertainty shock, the long end of the
yield curve decreases most, reflecting both an expected future decrease in short yields and a
temporary flight to quality. Gradually, the drop of long yields vanishes, whereas the decrease
in short yields appears instead, following the transmission of the increased uncertainty through
the real economy and monetary policy response.

The fiscal policy impact differs for the two types of shocks. The shock to the projected future
government spending works similarly to the uncertainty shock and therefore show that the
government spending shock causes a cautious response of economic agents. The cautious
response is two-fold. First, with government bond serving as a safe haven instrument, the latent
factor temporary decreases as a result of a flight to quality, pushing the long yields downwards.
Second, after some time, the real rate cycle starts decreasing instead. This demonstrates
the transition of the cautious response through the real economy, i.e. reflects the worsened
expectations of the agents on the future economic developments. Both these channels promote
the validity of the Barro-Ricardian equivalence and are in line with the findings of Ramey [2011],
who documents similar results for the relation between consumption and government spending.
The finding also offers insight into the interaction between the monetary and fiscal policy, where
the expected fiscal expansion also results in expected monetary expansion (or at least a lower
probability of tightening). In section 6 we further support the explanation of this decrease as
being caused by a cautious response.

The yield curve reacts oppositely to the increase in the relative government debt[14], which

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[14] Please note that we do not report here in the interest of space the results for the surprise shock in government
spending as the impulse responses are not significant and instead we focus on contemporaneous government debt.
Note: Each column represents an impulse in a single variable; the rows show responses on various horizons. The thin lines show the 68% confidence band. The impulses are normalized to one standard deviation of VAR innovations.

pushes the longer yields up while keeping the shorter yields mostly unchanged. This shock is transmitted through both the real rate cycle and the latent factor and reflects the government entering the fixed income market to obtain funds to finance the additional deficit. Simultaneously, an economic channel may be part of the explanation: the financing of the government spending is related to actual governmental purchases or tax cuts. The realized fiscal policy expansion may increase the aggregate demand in a way that would have been unexpected before the fiscal policy realization. This expansion consequently positively affects the real activity, which results in an increase in yields. These explanations and interaction between the expected government spending and changes in government debt are further supported in section 6.

Information about the importance of the particular channels can also be obtained using the forecast error variance decomposition (FEVD). FEVD for yields of selected maturities are shown in Figure 8. The macroeconomic factors (shocks to IPI, inflation cycle and real rate cycle) explain almost 80% of forecast error variance in short yields, which are cycle-driven, but less than 50% of the long yields, for which the latent factor shocks dominate. Regarding the fiscal variables, the short yields are relatively more affected by the government spending projections, whereas the importance of the government debt shocks gradually grows over the response horizon in case of the longer yields.

To understand the historic (realized) importance of the shocks, we use the VAR residuals as estimates of the innovations. We multiply the FEVD matrices by the residuals and take the sum of the responses over the response horizons. That means that in each period, the yield shock.
variation is explained by the innovations in the particular period plus responses to one-period-lagged innovations plus two-period-lagged innovations, etc. In this process, we are able to approximate the model-implied historical importance of the particular shocks. As the results show (Figure 8), the macroeconomic variables (IPI, inflation cycle and real rate cycle) explain approximately 50% (resp. 40%) of the historic variation of the short (long) yields. For short yields, the uncertainty variable was most important among the other variables, whereas in case of longer yields, the latent factor and government debt shocks dominated the other sources of yield variation.

Figure 8: Forecast Error and Historical Variance Decomposition of Selected Yields (in %)

Note: Each plot represents a share of the forecast error or historic variance of yield with particular maturity explained by the innovations to individual factors.

6. EXPLAINING THE EFFECTS

As a next step, we further explore the way fiscal policy shocks propagate through the real economy. To do so, we first enrich the baseline model with personal consumption expenditures (PCE) obtained from BEA (2018) transformed to log-differences. After such adjustment, the basic IRF scheme does not change significantly. Whereas the positive shock to projected government spending decreases the PCE, the positive shock to PCE shifts the yield curve upwards, especially for the short yields (for long yields, the effect is partially offset by a drop in term premia). Taking these effects together, at least part of the negative impact of the positive shock to government spending projections on yields may be explained by the cautious response of economic agents, who decrease the consumption expenditures and therefore lead to a drop in the real rate cycle.

As a second additional step in our analysis, we replace the projected government spending variable in the baseline model by the fiscal policy uncertainty index of Baker, Bloom, and Davis (2016) (FPU). FPU is constructed similarly to the general uncertainty variable included in the
model, i.e., it is newspaper-based, based on the news that includes the term "uncertainty" jointly with terms related to the fiscal policy. The result is shown in Figure 10 (dotted line). The IRFs of the baseline and adjusted models are very similar. Most importantly, the projected government spending shock and the FPU shock have a similar impact on the cyclical yield factors, which supports our argument about the cautious response of agents after an increase in projected government spending.

The only difference in the response is the increase of the latent factor after the FPU shock in contrast to the negative response in case of the government spending projections. It may be
argued that this difference reflects whether the U.S. Treasuries serve as a safe haven asset after the shock. As long as the future government spending is not related to increased uncertainty, the agents still seek U.S. Treasuries to hedge against possible negative developments in the economy. On the contrary, the U.S. Treasuries partially lose their position of safe haven in the environment of increased fiscal policy uncertainty and therefore their yields rise in the case of FPU shock.

Finally, the last additional step of the analysis offers the view on the interaction between the projected government spending and government debt shocks. So far, the results seemed as if these variables were unrelated. The reason lies in the span of the lags: the 5 month lag is insufficient to capture the longer-term transition between government spending projections and the impact of the spending realization on the economy and the government debt. Therefore, to show the longer-term interactions, we convert the data to quarterly and use four lags, i.e. the lags span over one year. The impulse responses remain roughly similar for the adjusted model (Figure 11).

The quarterly data model, however, estimates a significant increase in the government debt several quarters after the projected government spending shock (Figure 12). Given the positive

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The inclusion of higher number of lags or shift to quarterly data is not useful for the baseline model, since the model would either lose its robustness and identification or the quarterly data would lead to a loss of information about the short-term interactions. For the purpose of this additional analysis, we consider the comparison with the baseline results as a sufficient check that the adjusted results are roughly valid despite the mentioned weakness.
response of yields to the increase in government debt (reflecting both the effect of the shift in bond supply and the macroeconomic impact of the government spending after they are realized), the originally negative impact of the government spending on long-term yields turns positive after roughly 1.5 years (Figure 12, right plot). These results complete the full picture of the transition of the fiscal policy into the yield curve: after the initial drop caused by the cautious response of the agents, the yields increase when the government spending is realized, affects the economy and is financed.

![Figure 12: Impact of the Government Spending Shock in the Model with Quarterly Data](image)

7. Robustness Checks

We support the robustness of our results by performing multiple robustness checks (see Appendix III). First, we replace the IPI variable by the consumer sentiment index from the University of Michigan (2018). Second, we replace the projected government spending variable with the projected government budget balance. Third, we replace the used uncertainty index with the VIX index from CBOE (2018). In all cases, the adjusted models provide roughly similar results. Only the replacement of the spending by the budget balance leads to a lesser adjustment in the longer yields, which supports the discussion of the changing role of U.S. government bonds as a safe haven depending on the nature of shocks.

Finally, we also evaluate the effect of changing the number of lags in the model or adjusting the sample period. As representatives, we use lags 3, 5 (the baseline) and 7. The results are shown in Figure 16. We also calculate IRFs on samples excluding either the first or the last ten years (see Figure 17). The responses are roughly similar, although some of the responses are less significant or became significant at a different horizon.

The response of budget balance has correctly an opposite sign, since the positive budget imply relative lesser spending.
8. DEALING WITH THE LOWER BOUND ENVIRONMENT

An important recent part of the period is characterized by a binding lower bound on short yields. The lower bound is present because of the existence of a physical currency that bears zero yield (Krippner, 2013). Any negative yield can thus be avoided by transferring the funds to the physical currency. The lower bound does not need to be equal to zero due to transaction costs but is expected to be reasonably close to it. When yields hit the lower bound, their dynamics change (Krippner, 2015), since the distribution of the future yield movements becomes asymmetric: they can move only upwards, although the timing and speed of a future lift-off remain uncertain. This situation makes the results of the affine models, which are essentially Gaussian, invalid.

To check whether the lower bound affected our results, we replicate the analysis using a shadow-rate term structure model of Krippner (2013) further specified by Christensen and Rudebusch (2014). This model introduced the so called shadow rate. Technically, the shadow rate represents a yield of a hypothetical shadow bond. As proposed by Black (1995), the price of this shadow bond equals an observed bond price plus the price of a call bond option with a strike price equivalent to the lower bound yield (Christensen and Rudebusch, 2014).

The use of the shadow-rate model therefore allows us to model the yields without bias also close to the lower bound. However, the shadow-rate model is derived from the canonical Duffie and Kan (1996) affine model with the yield factors specified as latent \(^{17}\), i.e. the shadow model lacks the plausible properties of the model by Bauer and Rudebusch (2017) which we use as the baseline, whereas the values of term premia derived from this shadow-rate model may be subject to small sample bias (see the discussion in Bauer and Rudebusch, 2017). Therefore, the shadow-rate model is not suitable as the baseline model and serves as another robustness check while allowing us to discuss the consequence of the lower bound proximity explicitly.

The results of the shadow-rate model are shown in Figure 13. When compared to the baseline results (Figure 7), the responses are roughly similar in both sign and magnitude, with several differences. The shadow-rate responses do not die off quickly, since the shadow-rate yield factors include trends and therefore the system is close to non-stationary. Further, the response of the yield curve to a government debt shock is initially negative for the whole yield curve and after some lag only non-significantly positive for the longer yields. However, the cautious response to an increase in government spending projections is still present.

The option effect embedded in the shadow-rate model makes the relation of the yields and yield

\(^{17}\)The model is specified, in line with Christensen and Rudebusch (2014), as an affine dynamic Nelson Siegel model building on the framework of Christensen, Diebold, and Rudebusch (2011). Therefore, it uses three latent factors that can be specified as a level, a slope and a curvature of the yield curve.
factors non-linear (unlike the baseline model, see Equation 1). As a consequence, the response of the yields depends not only on the response of the yield factors but also on their level. This finding means that the steady state, to which the impulse-responses are related, also matters. If the shadow rate is significantly below the lower bound, the sensitivity of the observed yields to the shocks can be significantly reduced because the factor response is “consumed” by the option effect hidden in the shadow model. Since the sample period includes also the period when the yields were close to the lower bound, it is necessary to evaluate the extent to which the interpretation of the responses of the yields may be imprecise.

Therefore, apart from the full-sample estimation, we compute the responses of yields for two additional sets of steady states that reflect either pre-crisis or post-crisis average levels of yields (and yield factors). The results are shown in Figure 13. The two periods are divided by the Lehman Brothers collapse (September 2008). As the figure shows, the baseline estimation is not much influenced by the period of the lower bound proximity since the full-sample IRFs are close to the results when the lower bound period is excluded. However, when the steady state close to the lower bound is used, the option effect matters. Therefore, the responses are of a lesser magnitude in the low-yield environment. Technically, this result is caused by the option effect, which “consumes” a significant portion of the yield factor movements close to the lower bound. Economically, in a low-yield environment, unconventional monetary policy tools are adjusted first after shock, leaving monetary policy rates less affected. Therefore, this finding highlights that the response of yields to fiscal policy shocks could be suppressed over the recent period of the low yields, compared to the general results of our baseline analysis. With the yields leaving the lower bound environment, the sensitivity of the yield curve to the fiscal impulses can be expected to grow.

9. CONCLUDING REMARKS

The paper shows the impact of government expenditures on the yield curve in the US. The approach bridges multiple views from the literature. By following the fiscal foresight literature, we show the importance of the cautionary responses of the economic agents prior to fiscal policy realization which leads to a drop in yields. On the other hand, similarly to Dai and Philippon (2005), Ardagna, Caselli, and Lane (2007) and Laubach (2009), we find that the fiscal expansion shock identified at the moment when the fiscal policy is realized moves the yields upwards.

When explaining the transmission, we show that the cautious response has two effects. First, following the Ramey (2011) argument, we show that caution leads to a drop in personal consumption expenditures, which slows down the business cycle and leads to a decrease in the real rate. Second, we argue in line with the theoretical model of Horvath, Marsal, and Kaszab.
Note: Each column represents an impulse in a single variable; the rows show responses on various horizons. The response of yields is measured in basis points. The impulses are normalized to one standard deviation of VAR innovations.

In 2017, that the agents use government bonds as a safe haven instrument. We also document that the growth in the fiscal policy uncertainty, represented by the news-based indices of Baker, Bloom, and Davis (2016), follows the adjustment through the real channel, but not through the safe haven channel. This shows that the position of the U.S. bonds as a hedging instrument changes depending on the nature of the uncertainty.

By allowing for sufficient lag in the model, we also show the interaction and dynamic dimension of the precautionary response and the effect of the policy realization. More specifically, the initial drop in yields following expectation of the future policy changes is present roughly 1–1.5 year. Afterwards, the effect of the debt financing and the real effect of the policy implementation lead to a shift of the yields towards a temporary positive response. This finding has important implications for the interaction of government policy announcements, debt service and optimal timing of issuing new bonds to finance additional spending. Therefore, the finding presents an initial motivation for future research, which however requires granular data about the fiscal policy and therefore is left for the future.

Finally, we show that the impact of government spending on bond yields is of lower magnitude in the low-yield environment, which constrains conventional monetary policy (see Buiter 2009).
and Swanson and Williams [2014]. The lower bound proximity could be an important part of the explanation of the recent interaction between fiscal and monetary policy. However, the interaction between monetary and fiscal policy at the lower bound requires deeper evaluation, which we also leave for future research.

We support our results with robustness checks. The responses are similar for both government spending and fiscal budget balance shocks. The results are also robust to including differently specified control variables, to changes in the sample period and to the lag selection.

The identified decrease in yields before the fiscal policy realization, following the saving motives, has important consequences. For example, introducing expansionary fiscal policy steps may temporarily decrease the costs of the debt service, with the consequence of overoptimistic future debt service projections and, therefore, the further extension of the planned fiscal expansion.

Finally, the identified temporary decrease in the yields ahead of the realization of the fiscal stimulus goes against an increase in credit risk premia, which should be related to the shift in the sustainability of government debt. Further research on this topic is however needed. A possible next step would be to extend the analysis to a panel of countries, which would allow us to observe in greater detail the relationship between cautious motives at the moment the fiscal policy becomes expected, the bond supply and real effect after the policy realization and the overall impact of the government indebtedness on the yields.
REFERENCES


APPENDICES

APPENDIX I: TERM STRUCTURE MODEL REPRESENTATION AND PARAMETERS

The term structure model follows the illustrative model of Bauer and Rudebusch (2017). Following the basic affine term structure framework of Duffie and Kan (1996), its core lies in several yield factors that govern evolution of yields. The five factors used in the model are the equilibrium inflation $\pi^*$, the inflation cycle $\pi^c$, the equilibrium real rate $r^*$, the real rate cycle $r^c$ and the latent variable determining the price of risk $f$. Under the data-generating $P$ measure, the equilibrium factors are assumed to follow random walk, whereas the cyclical factors and the price of risk factor are assumed to follow stationary AR(1) process. The joint dynamics under the $P$-measure can be therefore written as a VAR(1) process:

$$X_t = \Phi X_{t-1} + \Sigma \epsilon_t$$  \hspace{1cm} (3)

where $X_t = [\pi_t^*, \pi_t^c, r_t^*, r_t^c, f_t]^\prime$ is the vector of the yield factors, $\Phi = diag([1, \phi_{\pi}, 1, \phi_{r}, \phi_{f}]^\prime)$ is the VAR(1) transition matrix with non-unit diagonal elements assumed to be less than one in the absolute value, $\epsilon_t = [\epsilon_{t,\pi^*}, \epsilon_{t,\pi^c}, \epsilon_{t,r^*}, \epsilon_{t,r^c}, \epsilon_{t,f}]^\prime \sim N(0, I_5)$ are iid random disturbances and $\Sigma$ is a diagonal matrix of standard deviations of the random disturbances.

Following the usual approach as in Ang and Piazzesi (2003), the price of a $\tau$-maturity nominal bond $P_t(\tau)$ in time $t$ (or any other asset yielding no dividend) is under the risk-neutral $Q$ measure equal to the expected value of one-period-ahead value discounted by the risk-free real interest rate and the future inflation (all under the expectations):

$$P_t(\tau) = E^Q_t [\exp (-r_t - \pi_{t+1}) P_{t+1}(\tau - 1)]$$  \hspace{1cm} (4)

To obtain the bond prices in relation to the $P$-measure dynamics, the Radon-Nikodym derivative $\psi_{t+1}$ is used to convert the $Q$-measure into the $P$-measure, since for any random variable $V_{t+1}$ the following holds (Ang and Piazzesi, 2003):

$$E^Q_t [V_{t+1}] = E^P_t \left[ \frac{\psi_{t+1} V_{t+1}}{\psi_t} \right]$$

The existence of $\psi_{t+1}$ represents the assumption of no-arbitrage, as it translates the price of risk into the linkage between the two measures. Assuming it follows the log-normal process,
the Equation 4 then transforms to:

\[ P_t(\tau) = E_t^P \left[ \exp \left( -r_t - \pi_{t+1} \frac{\psi_{t+1}}{\psi_t} P_{t+1}(\tau - 1) \right) \right] \]

\[ = E_t^P \left[ \exp \left( -r_t - \pi_{t+1} - \lambda'_t \lambda_t / 2 - \lambda'_t \epsilon_{t+1} \right) P_{t+1}(\tau - 1) \right] \]

and therefore:

\[ P_t(\tau) = E_t^P \left[ m_{t+1} P_{t+1}(\tau - 1) \right] \quad (5) \]

where \( \lambda_t \) is a stochastic vector of market prices of risk associated to the risks represented by the random disturbances \( \epsilon_{t+1} \). Put differently, the Radon-Nikodym derivative process gathers the information about the stochastic risk premia, which were originally included in the risk-neutral measure \( Q \), but after the transformation, they are explicitly expressed in terms of the \( \lambda_t \) process in the pricing equation under the \( P \)-measure. The whole discount factor including the market price of risk is called as the stochastic discount factor, or pricing kernel, \( m_{t+1} \).

The pricing kernel can be further expanded to build the affine model. The market price of risk \( \lambda_t \) is assumed to be an affine transformation of the yield factors. Following the restrictions imposed by Bauer and Rudebusch (2017), this transformation makes the latent factor \( f_t \) the only source of variation in \( \lambda_t \), however the shocks to \( f_t \) (i.e. \( \epsilon_{f,t} \)) are not priced:

\[ \lambda_t = \lambda_0 + \lambda_1 \left[ \pi^*, \pi^c, r^*, r^c, f \right]'_t \]

\[ = \begin{bmatrix} \lambda_{0, \pi^*} \\ \lambda_{0, \pi^c} \\ \lambda_{0, r^*} \\ \lambda_{0, r^c} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & \lambda_{1, \pi^*} \\ 0 & 0 & 0 & \lambda_{1, \pi^c} \\ 0 & 0 & 0 & \lambda_{1, r^*} \\ 0 & 0 & 0 & \lambda_{1, r^c} \\ 0 \end{bmatrix} \begin{bmatrix} \pi^* \\ \pi^c \\ r^* \\ r^c \\ f \end{bmatrix}_t \quad (6) \]

\[ + \begin{bmatrix} \lambda_{0, \pi^*} \\ \lambda_{0, \pi^c} \\ \lambda_{0, r^*} \\ \lambda_{0, r^c} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & \lambda_{1, \pi^*} \\ 0 & 0 & 0 & \lambda_{1, \pi^c} \\ 0 & 0 & 0 & \lambda_{1, r^*} \\ 0 & 0 & 0 & \lambda_{1, r^c} \\ 0 \end{bmatrix} \begin{bmatrix} \pi^* \\ \pi^c \\ r^* \\ r^c \\ f \end{bmatrix}_t \]

where \( \lambda_{0,*} \) and \( \lambda_{1,*} \) are scalar parameters.

Furthermore, the rest of the pricing kernel can be also directly expressed in terms of the yield factors by putting \( r_t = r_t^* + r_t^c \) and \( \pi_{t+1} = \pi_t^* + \pi_t^c + \epsilon_{t+1} \) according to Bauer and Rudebusch (2017), where \( \epsilon_{t+1} \sim N(0, \sigma_e^2) \) is the inflation noise component. By plugging in, the pricing kernel adjusts to:

\[ m_{t+1} = \exp \left( -r_t^* - r_t^c - \pi_t^* - \pi_t^c - \epsilon_{t+1} - \lambda_t' \lambda_t / 2 - \lambda_t' \epsilon_{t+1} \right) \]

The pricing kernel is therefore exponentially affine in the yield factors, whose innovations are assumed to be normally distributed. Returning to the pricing equation, plugging in the unit value at maturity and using the properties of log-normal distribution and the fact that the risk in \( \epsilon_{t+1} \)
is not priced (Bauer and Rudebusch [2017], the value of a one-period bond equals to:

\[ P_t (1) = E_t^P [m_{t+1} P_{t+1} (0)] \]

\[ = E_t^P \left[ \exp \left( -r_t^* - r_t^C - \pi_t^e - \pi_t^C - e_{t+1} - \lambda_t^e \epsilon_t + \frac{\lambda_t^C}{2} \right) \right] \]

\[ = \exp \left( - \left[ r_t^* + r_t^C + \pi_t^e + \pi_t^C + \lambda_t^e \epsilon_t + \frac{\lambda_t^C}{2} \right] \right) E_t^P \left[ \exp \left( -e_{t+1} - \lambda_t^e \epsilon_{t+1} \right) \right] \]

\[ = \exp \left( - \left[ r_t^* + r_t^C + \pi_t^e + \pi_t^C + \lambda_t^e \epsilon_t + \frac{\lambda_t^C}{2} \right] + Var_t^P \left[ -e_{t+1} - \lambda_t^e \epsilon_{t+1} \right] / 2 \right) \]

\[ = \exp \left( - \left[ r_t^* + r_t^C - \pi_t^e - \pi_t^C - \lambda_t^e \epsilon_t + \frac{\lambda_t^C}{2} + Var_t^P \left[ \lambda_t^e \epsilon_{t+1} \right] / 2 \right] \right) \]

\[ = \exp \left( -r_t^* - r_t^C - \pi_t^e - \pi_t^C + \sigma_t^2 / 2 \right) \]

The one-period bond is priced using the short-term yield \( y_t (1) \):

\[ P_t (1) = \exp (-y_t (1)) = \exp \left( -r_t^* - r_t^C - \pi_t^e - \pi_t^C + \sigma_t^2 / 2 \right) \]

\[ y_t (1) = r_t^* + r_t^C + \pi_t^e + \pi_t^C - \frac{\sigma_t^2}{2} = A_1 + B_1 X_t \]

The short-term yield is therefore a sum of the equilibrium and cyclical factors adjusted by the convexity effect of the inflation noise risk. The short-term yield is an affine transformation of the yield factors \( X_t \) with the constant \( A_1 = -\sigma_t^2 / 2 \) and the loadings \( B_1 = [1, 1, 1, 1] \).

After plugging \( P_t (1) \) into the pricing equation (Equation 5), the value of \( P_t (2) \) may be equivalently obtained, and following this iterative procedure may be used to obtain the whole term structure of bond prices and therefore yields. For \( \tau > 1 \), the yield loadings \( B_{\tau} \) are different to a vector of ones, since the market price does not disappear anymore. However, can be obtained in a form of recursion, as shown by [Ang and Piazzesi (2003)]:

\[ A_{\tau+1} = \frac{\tau}{\tau + 1} A_{\tau} + \frac{\tau}{\tau + 1} B_{\tau} (-\Sigma \lambda_0) - \frac{\tau^2}{2 (\tau + 1)} B_{\tau} \Sigma \Sigma' B_{\tau} + \frac{1}{\tau + 1} A_1 \]

\[ B_{\tau+1}' = \frac{\tau}{\tau + 1} B_{\tau}' (\Phi - \Sigma \lambda_1) + \frac{1}{\tau + 1} B_1' \]

Bauer and Rudebusch [2017] show that their restrictions on the matrices \( \Phi, \Sigma, \lambda_0 \) and \( \lambda_1 \) yield convenient simplification of these recursive formulas:

\[ y_t (\tau) = A_{\tau} + B_{\tau}' X_t = r_t^* + \frac{1 - \phi_t^e}{\tau (1 - \phi_t^e)} r_t^C + \pi_t^e + \frac{1 - \phi_t^C}{\tau (1 - \phi_t^C)} \pi_t^C + A_{\tau} + B_{\tau}' f_t \]

where \( B_{\tau}' \) is the last element of \( B_{\tau} \).

Such yield equation has plausible properties: yields of all maturities include the full amount of the equilibrium factors \( r_t^* \) and \( \pi_t^C \). The loadings on the cyclical factors \( r_t^C \) and \( \pi_t^C \) are decreasing with increasing maturity, as the mean expectations about the future yields give lesser weight on the present cyclical deviation. Finally, the convexity effect \( A_{\tau} \) and the time-varying risk premia
B$^f_i$ are more important for longer maturities and are therefore responsible for the deviation of the longer-yields from the pure expectations about future evolution of short yields.
APPENDIX II: VAR MODEL LAG ANALYSIS

Figure 14: Baseline VAR Information Criteria

- AIC (left axis)
- BIC (right axis)
APPENDIX III: SENSITIVITY CHECKS

Figure 15: IRFs Sensitivity Checks: Different Choice of Variables

Note: Each column represents an impulse in a single variable; the rows show responses on various horizons. The impulses are normalized to one standard deviation of VAR innovations.
Figure 16: IRFs Sensitivity Checks: Different Lag Selection

Note: Each column represents an impulse in a single variable; the rows show responses on various horizons. The impulses are normalized to one standard deviation of VAR innovations.
Figure 17: IRFs Sensitivity Checks: Trimmed Period

Note: Each column represents an impulse in a single variable; the rows show responses on various horizons. The impulses are normalized to one standard deviation of VAR innovations.