THEORETICAL VALUE OF THE FORWARD RATE FOR A FOREIGN CURRENCY AND RISK-FREE ARBITRAGE

prof. RNDr. Vincent Šoltés, CSc., Faculty of Economics, Technical University of Košice

In the works of various authors (for example [1] and [3]) there is presented the relationship for calculating the theoretical value of the forward rate for a foreign currency. This relationship may be derived (using the international Fisher Effect), though under ideal conditions, namely that the interest rates on deposits and loans are equal and also that exchange rates for the purchase and sale of foreign currency are equal. In practice, however, not one of these conditions applies and the differences are quite large. For example on 6 February 2004 Poštová banka quoted the exchange rate for the purchase – sale (the aspect of a bank) of American dollars as 30.91 – 33.63. From the client's aspect there is a mirror image, i.e. from our aspect (the client) is $SR_S = 30.91$ (we can sell at this rate) and $SR_B = 33.63$ (we can buy at this rate). It is logical that always for the client $SR_S < SR_B$ i.e. the bank buys from us at a rate lower than that at which it sells to us.

A forward for the purchase, or sale, of a foreign currency is an agreement for the purchase, or sale, of a foreign currency in the future at a forward rate agreed today.

The buyer in a forward for a foreign currency undertakes to buy the foreign currency at an agreed date in the future at a forward rate agreed at the present FR_B i.e., regardless of what the spot rate is at the given time. Clearly, the buyer in a forward for a foreign currency expects that the foreign currency will be more expensive in the future than the agreed FR_B.

Conversely, the seller in the forward for the foreign currency undertakes to sell the foreign currency at the agreed rate. The seller thus has an expectation as to the foreign currency's development that is opposite to that of the buyer.

Even if the theoretical forward rate could be a matter of agreement between the buyer and the seller, in actuality certain limitations exist, which these forward rates must fulfil; otherwise a risk-free arbitrage situation would exist, i.e. there would be the opportunity to earn any amount risk-free.

The aim of this article is to define these limits and, in the case of their non-fulfilment, give instructions for risk-free arbitrage.

Derivation of the condition for the forward rate FR_B for buying a foreign currency

Let us assume that we have the possibility to conclude a forward for purchasing a foreign currency at the forward rate FR_B and with the maturity t in days (the purchase is made in t days' time). Let us also assume that the nominal interest rate for a loan in the foreign currency with a maturity of t days is r_{FL}, the spot rate for selling the foreign currency is SR_S and the nominal interest rate on a term deposit for t days in the domestic currency is r_{DD}. From the following we arrive at the condition for the forward rate FR_B for buying a foreign currency and concurrently also a guide to arbitrage in the case of the condition not being fulfilled.

a) We take out a loan in the amount L in the foreign currency, with a maturity of t days, thus at the maturity date we will have to repay

$$L\left(1+\frac{t}{360}\cdot\frac{r_{FL}}{100}\right) = L\left(\frac{36000 + t\,.\,r_{FL}}{36000}\right) \quad \text{of the foreign currency.}$$

b) We sell the amount L of the foreign currency at the spot rate SR_S and deposit the amount of domestic currency obtained in a term-deposit account for t days, following which we will have

$$L\,.\,SR_S\left(\frac{36000 + t\,.\,r_{DD}}{36000}\right) \quad \text{of the domestic currency.}$$

c) We conclude a forward for buying the foreign currency at the forward rate FR_B, where the size of the contract is

$$\frac{L\,.\,SR_S\left(\frac{36000 + t\,.\,r_{DD}}{36000}\right)}{FR_B}.$$

If risk-free arbitrage is not to exist then the condition must be fulfilled that

$$L\left(\frac{36000 + t\,.\,r_{FL}}{36000}\right) \geq \frac{L\,.\,SR_S\left(\frac{36000 + t\,.\,r_{DD}}{36000}\right)}{FR_B},$$

from which we arrive at

$$FR_B \geq SR_S\left(\frac{36000 + t\,.\,r_{DD}}{36000 + t\,.\,r_{FL}}\right), \tag{1}$$

which is the condition for the forward rate for buying the foreign currency from the aspect of the client (from the bank's aspect this is a sale rate).

Note 1. If the time t up to the maturity of the forward is in years and r is the interest rate in the tenth number, then the condition has the form:

$$FR_B \geq SR_S \left(\frac{1 + t \,.\, r_{DD}}{1 + t \,.\, r_{FL}} \right). \qquad (2)$$

Example 1. The possibility exists on the market to conclude a three-month forward for buying USD at the forward rate $FR_B = 31.55$. Let us see whether it is possible to earn at least USD 2000 risk-free, where the nominal interest rate on the three-month loan in USD is 2.3%, the nominal interest rate on the three-month term-deposit in SKK is 3.9% and the current USD sale rate is 31.50.

Solution. Since in our case

$$SR_S \left(\frac{36000 + t \,.\, r_{DD}}{36000 + t \,.\, r_{FL}} \right) = 31{,}50 \left(\frac{36000 + 90 \,.\, 3{,}9}{36000 + 90 \,.\, 2{,}3} \right) =$$

$$= 31{,}625 > 31{,}55,$$

condition (1) is not fulfilled. The forward rate is set incorrectly. Therefore a risk-free arbitrage opportunity exists. The procedure is as follows:

- we take out a three-month loan in USD in the amount L (we shall specify this in the conclusion) with a nominal interest rate of 2.3%,
- we sell the money at the spot rate 31.50 and deposit the proceeds in a three-month term-deposit account with a nominal interest rate of 3.9%,
- we conclude a three month-forward rate agreement for buying USD at the forward rate FR_B = 31.55 for the amount

$$\frac{L \,.\, 31{,}50 \left(\dfrac{36000 + 90 \,.\, 3{,}9}{36000} \right)}{31{,}55} = 1{,}008149 \,.\, L$$

Since we want to earn at least USD 2000 the condition must apply that

$$1{,}008149\, L - L \left(\frac{36000 + 90 \,.\, 2{,}3}{36000} \right) \geq 2000\,,$$

from which we arrive at

$L \geq 833\,681$. If we were to take out a three-month loan in this amount then we would earn risk-free USD 2000.

Derivation of the condition for the forward rate FR_S for selling a foreign currency

Let us now assume that we have the possibility to conclude a forward for selling a foreign currency at the forward rate FR_S and with the maturity t in days. Let us also assume that the nominal interest rate for a loan in the domestic currency with a maturity of t days is r_{DL}, the spot rate for buying the foreign currency is SR_B and the nominal interest rate on a term deposit for t days in the foreign currency is r_{FD}. In this case we proceed as follows:

a) We take out a loan with a maturity of t days, in the amount L in the domestic currency.

b) From the funds obtained we buy foreign currency and deposit it in a term-deposit account.

c) We conclude a forward for selling the foreign currency at the forward rate FR_S, where the size of the contract is

$$\frac{L}{SR_B} \left(\frac{36000 + t \,.\, r_{FD}}{36000} \right).$$

If risk-free arbitrage is not to exist then the condition must be fulfilled that

$$\frac{L}{SR_B} \left(\frac{36000 + t \,.\, r_{FD}}{36000} \right) .\, FR_S \leq L \left(\frac{36000 + t \,.\, r_{DL}}{36000} \right),$$

thus

$$FR_S \leq SR_B \left(\frac{36000 + t \,.\, r_{DL}}{36000 + t \,.\, r_{FD}} \right), \qquad (3)$$

which is the condition sought for the forward rate for a sale.

Note 2. If the time t up to the maturity of the forward is in years and r is the interest rate in tenth number, then the condition has the form:

$$FR_S \leq SR_B \left(\frac{1 + t \,.\, r_{DL}}{1 + t \,.\, r_{FD}} \right). \qquad (4)$$

Note 3. In the ideal (even if in practice unrealistic case), where rates for the purchase and sale are equal and also interest rates on deposits and loans are equal, then from the relationships (1) and (3), or (2) and (4) we arrive at the fact that

$$FR = SR \left(\frac{36000 + t \,.\, r_D}{36000 + t \,.\, r_F} \right), \qquad \text{or}$$

$$FR = SR \left(\frac{1 + t \,.\, r_D}{1 + t \,.\, r_F} \right),$$

which is the relationship stated in the works [1], [3], or [4].

References:

1. Markovič, P.: Financial Engineering, IURA EDITION, Bratislava, 2001 (in Slovak)
2. Šoltés, V.: Financial derivatives, Košická tlačiarenská, a.s., Košice, 2002 (in Slovak)
3. Vlachynský, K.: Financial Engineering, IURA EDITION, Bratislava, 2001 (in Slovak)
4. Vlachynský, K: Financial derivatives and their use in our practice, Biatec, No. 12, 1996 (in Slovak)