



# The equilibrium interest rate – theoretical concepts and applications

Part 1

Ing. Michal Benčík  
Národná banka Slovenska

*The article presents several theoretical equilibrium interest rate concepts; it applies some of them and assesses the influence of the introduction of euro on this issue.*

*We present calculations of the neutral interest rate for Slovakia and for the euro area, as well as an approximation of the long-term equilibrium interest rate. The neutral interest rate is determined by the IS curve and the Phillips curve. It takes values close to zero, which are, however, distorted by a general excess of saving over investment. The long-term interest rate is determined directly by means of a modified Ramsey-Keynes rule and takes values around 2.5%. After Slovakia's accession to the monetary union, there will be a discrepancy between the neutral interest rate in the euro area and in Slovakia. It will be probably necessary to offset the expansive influence of interest rates by other economic policy instruments.*

## 1 INTRODUCTION

The interest rate is a phenomenon, which we encounter in various macroeconomic and microeconomic contexts. Interest is paid on – both retail and interbank – deposits and loans, profit maximization of firms implies a connection between the interest rate and capital productivity, which, along with a variable effectiveness of the individual projects, leads to investment demand and total demand with interest rates. Under certain circumstances, the consumers can decide between current and future consumption based on the interest rate. The central bank can influence the production gap and indirectly also inflation by announcing interest rates.

Economic theory considers the national economy a system that exhibits the dynamic stability property in the sense that there is a state of equilibrium, to which it converges in the absence of shocks. In most cases, the state is characterized by the non-existence of internal conflicts and optimality. At an abstract level, the equilibrium interest rate can be defined as the interest rate consistent with the state of equilibrium. The definitions of the equilibrium itself can differ and the corresponding equilibrium interest rates will also differ accordingly. Particularly important for central banks is the concept of an equilibrium interest rate corresponding to the dual mandate of maintaining inflation at a target level and simultaneously minimizing real disequilibria.

In practice, the national economy is permanently exposed to various changes. If, theoretically, there were no changes and shocks, the equilibrium interest rate should be constant. In practice,

the usefulness of such a concept is low. This paper will thus deal with estimates of a variable equilibrium interest rate, because it is influenced by various technological, psychological and institutional factors.

New trends of application of microfoundations in the analysis of the economy are represented by the dynamic stochastic general equilibrium (DSGE) models, a generalization of the so-called real business cycle (RBC) models. Those models explicitly allow for the variability of the interest rate and its reaction to real shocks. Because the interest rate also contains an explicit random component in those models, the equilibrium interest rate can be identified with the interest rate component corresponding to macroeconomic equilibrium.

## 2 THEORETICAL CONCEPTS

### 2.1 General knowledge

An interest is a payment that the debtor pays to the creditor for lending money over for a certain period of time. We can think of it as a rent for money. The ratio of the interest to the amount lent (the principal) is called the interest rate. There are various ways to calculate interests. In this paper, we assume compound annual interest, which is the most common way of interest calculation. We start our analysis by looking at nominal interest rates, because interest rates are more frequently defined as nominal in everyday life (for deposits and loans). If interest is added to principal  $A_0$  at time  $t$  the total debt  $A$  has the following value for a stable nominal interest rate



1 This is an approximation. The exact formula is  $1 + it = (1 + rt)(1 + E(\pi_{t+1}))$   
2 Details on the issue can be found in Munro (2007).

$$A_t = A_0(1 + i)^t,$$

where  $t = 1, 2, 3$  etc. from the period onwards, in which the original liability arose.

Various unfavorable circumstances can arise between the time where the creditor lends money to the debtor and the time where the creditor gets the money back from the debtor: the money can lose a part of its value as a result of inflation, the situation of the debtor can deteriorate so far that he is not able to repay his debt, or the creditor might need the money that he has lent and he will not have the money. In practice, nominal interest rates are the higher, the more likely these events are. If the economic agents are rational and perfectly informed, the conditions of the loan will take into account not only the risk of inflation, but also the risk of non-payment for the given debtor and the price of giving up liquidity.

If the creditor lends money cautiously (thereby minimizing the risk of default on the part of the debtor), the growth of prices – inflation – is the most important risk factor. Ideally, a creditor not suffering from money illusion (i.e. he can recognize the impact of inflation on the nominal prices of goods) will take into account in the demanded interest the expected decrease in the value of his money between the time of lending the money and the time of its full repayment. The logic of this statement is described by the Fisher equation, which defines the nominal interest rate as the sum of <sup>1</sup> the so-called real interest rate and the expected inflation in the following period:

$$i_t = r_t + E(\pi_{t+1})$$

The real interest rate thus represents a fundamental valuation of temporary provision of capital (money) corresponding to a price level constant in time. It is also obvious from the above relation that if inflationary expectations change, nominal interest rates have to change aliquotly at a constant real interest rate. The real interest rate concept is irreplaceable in the research into the mutual relations of inflation, because assuming that the creditors are rational, inflation and nominal interest rates influence each other. For similar reasons, the real interest rate is used in broader economic analyses.

Expected inflation is an unobservable quantity. In an ex-post analysis, it can be replaced by the actual rate of inflation in the following period, which is equivalent to assuming rational expectations. Theoretically less satisfactory, but easier to apply, is the assumption of adaptive expectations, which replaces expected inflation in the future by actual inflation in the present, so that  $i_t = r_t + \pi_t$ . We use this relation in this paper. Adjustment for inflation is very important, because when there is increased inflation over a long period of time, economic agents (particularly businesses) recognize the actual value of money, stop suffering from money illusion and accept increased nominal rates. Therefore, investment as the main link

between the interest rates and the real economy is considered a function of the real interest rates, as standard.

In most cases, the risk of default is relevant mainly when lending to natural persons and businesses. It depends primarily on the type of the debtor and the overall macroeconomic environment. On the other hand, the interest rates, at which central banks sterilize free funds, do not include this aspect, because central banks are, by definition, always solvent. In this paper, we are going to focus on the interest rates on the interbank market, where the risk of default is greater than zero, but quite low, so that we neglect it. If we analyzed, for example, interest rates on loans to non-financial entities, this aspect would have a greater impact on the results. On the other hand, the fact that most economic agents encounter primarily such rates speaks in favor of retail interest rates.

Under normal circumstances, the price of giving up liquidity is the higher the longer the period is, over which the money lent is not available (e.g. interest rates on time deposits are higher than those on demand deposits – but retail rates are also influenced by other factors). In our case, we assume that the price of giving up liquidity is directly a part of the equilibrium interest rate.

## 2.2 The forerunner of the equilibrium interest rate

As early as in the Middle Ages, some scholastic scholars (particularly Hostiensis) held in assessing usury that a “fair” return on money lent equals the foregone profits or opportunity cost. When the principal is repaid, the creditor requires from the debtor an additional return, which is equivalent to the return he would achieve, if he used his property himself.<sup>2</sup>

This criterion leads to a certain value of “fair return”, which can be understood as the equilibrium interest rate. Let us consider a production financed by a security with a unit nominal value, which has a net return  $d$  over a certain maturity period. The return equals the net profit from the production at hand. Let us assume that the return can be reinvested without restrictions (the factor supply is indefinitely elastic). In such a case, the production at hand will be growing at a growth rate equal to  $d$ . Let us assume that it is possible to invest alternatively in a fixed-interest security with the same maturity period as the previous security and with the interest rate  $r$ . Assuming risk neutrality, the investors will be indifferent between those securities, if  $d$  equals  $r$ , which represents a certain type of equilibrium. Because in our case  $d$  is also the potential growth rate, the “fair” return, according to this theory, should be equal to the long-term growth rate. Thus the growth rate of potential output can approximate the equilibrium interest rate in simplified concepts.

## 2.3 Wicksell's definition

A classical definition of the equilibrium interest rate has been provided by Wicksell (1898); in his



terminology, the equilibrium interest rate is the „natural“ interest rate. The market interest rate fluctuates around a natural level. Alternatively, he called the natural interest rate the normal or real interest rate.

The natural interest rate fulfills the following three criteria:

It represents the rate, at which investment goods would be lent in kind. The concept of the equilibrium interest rate as opportunity costs fulfills this requirement under certain circumstances. Under this criterion, disequilibrium either restricts investment (if the market interest rate is higher than the natural interest rate) or it enables to implement high-return projects (if the market interest rate is too low).

In addition, Wicksell requires that the natural interest rate be neutral for the prices of commodities (he assumes the natural interest rate is linked to inflation). If the market rate is lower than the natural rate, the demand for loans is higher than saving, the money supply expands by multiplication by means of bank accounts and the prices increase. Similarly to the view of the Austrian School, the expansion starts at high natural interest rates. By pointing to the connection between disequilibrium in the area of interest rates and inflation, Wicksell anticipated the theoretical foundations of modern monetary policy well in advance.<sup>3</sup>

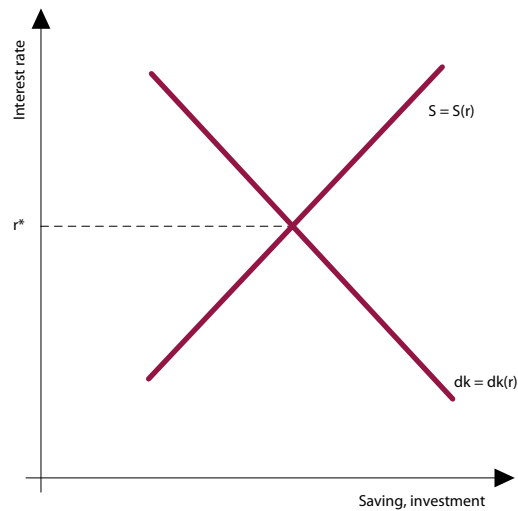
Moreover, the natural interest rate is supposed to coordinate saving and investment. This function was also important for preventing the effective money supply from increasing in the manner described in previous paragraph. This requirement is related to the general equilibrium, because saving depends on decisions of the households and investment on decisions of firms. According to Blaug (1986), the aim of the whole work of Wicksell was to connect the Austrian School (Böhm-Bawerk), the general equilibrium theory (Walras) and marginalist classical theory (Ricardo).

This concept is of normative nature. It is defined mostly verbally and is not suitable for practical quantification, unless we consider averaging or one-dimensional smoothing of time series.<sup>4</sup> In line with Wicksell's requirement, Bernhardsen determines the interest rate for Norway from the equilibrium interest rate for the euro area, which theoretically equates local saving and local investment. Archibald and Hunter proceed analogously for New Zealand. They consider the rate equating saving and investment in the USA the basis. In both cases the result is adjusted for the risk premium for the given country or for the impact of capital flows. The cited works do not contain a calculation of foreign interest rates; those are taken over from other works.

#### 2.4 Neutral interest rate

The first half of the 20<sup>th</sup> century brought a fast development of economic statistics, macroeconomics and economic modeling. I consider the following progressive events particularly important in the context of this paper:

Chart 1 Saving, investment and interest rate



The equilibrium interest rate in domestic economy

Source: Author based on Bernhardsen (2005).

- establishing of the system of national accounts including the definition of gross national income and its components (compared to older data, they constitute a new generation of economic indicators suitable for systematic modeling),
- postulation of the Fisher equation splitting the interest rate into the real interest rate and the expected rate of inflation,
- Keynes' and Kahn's contributions to the economic science, particularly the IS-LM model, which leads to simple relations between macroeconomic quantities, which can be used in medium-term economic models.

In addition to these instruments, contributions of Philips and Phelps related to the relations of the economic cycle and inflation are irreplaceable for our work.

By means of those concepts, we can define the neutral interest rate as the real interest rate, at which inflation is stable<sup>5</sup> and the production gap equals zero. That interest rate very often appears in monetary policy deliberations (Bernhardsen 2005). Such an interest rate occurs in the Taylor rule (Taylor 1993). The Taylor rule describes the reaction function of the central bank, which adjusts the announced interest rate according to imbalances in the economy ( $i$  means the actual nominal interest rate,  $r^*$  is the neutral real interest rate,  $\pi^*$  is the target inflation rate,  $\pi$  is the actual rate of inflation,  $y$  is the logarithm of the actual product and  $y^*$  is the logarithm of the potential product). As a rule, it is required that the sensitivity of the announced interest rate to an inflation imbalance be higher than one, so that the real interest rate influencing the demand changes in the right direction in the case of increased inflation. Using standard weights, the rule takes the following form:

$$i = r^* + \pi^* + 0,5 (y - y^*) + 1,5 (\pi - \pi^*)$$

3 However, his analysis was not complete, because he did not distinguish between the nominal and the real interest rate yet.

4 Estimates of the real equilibrium interest rate can be also achieved in those ways. Averaging is based on the assumption that in the long run the actual values of real interest rates fluctuate just around the equilibrium interest rate, which is constant (which de-facto does not contain any economic theory and is quite restrictive). Onedimensional filtering of a real interest rate time series, e.g. by means of a HP filter, introduces the quite unreal assumption that the nominal interest rates and inflation have a common cyclical component.

5 Equals a target value.



The Taylor rule works in accordance with anti-cyclical economic policy – if the economy overheats, the real interest rates increase and cause the demand to decline, and if the economy works at a low degree of capacity utilization, the interest rates decrease and stimulate the demand. It is obvious that the estimate of the Taylor rule will provide a constant term as the basis for the simplest estimate of the neutral interest rate. The interest rate, however, does not have to be constant and, in addition, it is not clear whether the central bank consistently followed just the used form of the Taylor rule in the period under review. Assuming that an estimate of the Taylor rule is possible, several studies dealing with the estimate of a non-constant equilibrium interest rate are motivated by the distortion in the estimate of the Taylor rule (Trehan and Wu, 2004). In practice, the opposite procedure, too, is used in deliberations on the actual interest rate: starting from some value, which is considered an equilibrium value, the value is increased by adding the target inflation and it is adjusted downwards or upwards depending on the imbalances in the inflationary development and in the real economy.

To obtain the equilibrium interest rate defined in this way, it is possible to use the Kalman filter, a DSGE model, a structural econometric model or a VAR model. In calculations using the Kalman filter, we explicitly start from estimates of the IS curve and the Philips curve. Crespo Cuaresma et al. (2004) decompose the interest rate together with other variables by means of the Kalman filter to a trend component, cyclical component and a random component, and they use trigonometric functions to define the cycle.

DSGE models work with variables in the form of deviations from the trend. In a typical DSGE model (e.g. Kanczuk), the interest rate is one of the stochastic variables that introduce shocks into the model. It has four components: a constant long-term level (similarly to the Ramsey model), an inertial component, a component corresponding to the reaction to real shocks and the actual random component. In a broader sense, the equilibrium interest rate can be defined as the sum of the first, second and last component. An equilibrium rate determined in this way is a function of the Solow residual.

When using a structural econometric model with an exogenous interest rate, we set the target variables (the output and inflation) and search for the values of instrumental variables (the interest rate) to achieve them. Usually the number of needed instrumental variables equals the number of targets, so that the equilibrium values of interest rates are part of some equilibrium scenario and are conditional on a certain trajectory of another exogenous variable or other exogenous variables. If we define the shocks in a structural VAR model in an appropriate way, then after a decomposition according to Blanchard and Quah, we can calculate a similarly defined equilibrium interest rate by a cumulation of structural shocks (Brzoza-Brzezina, 2002).

## 2.5 Equilibrium interest rate determined by the real interest rate parity

The definition of the neutral interest rate does not take into account the international aspects of the financial market. The possibility of arbitrage in the foreign exchange market, however, implies that when there is free movement of capital, the short-term capital will maximize the return composed of the interest return and exchange rate differentials. If we assume two countries, the expected change in the real exchange rate equals the difference between real interest rates increased by the risk premium:

$$E(s_{t+1}) = r_t - r_t^f - rp$$

By adjusting this condition and putting in an exogenous value for the equilibrium interest rate abroad  $r^{f*}$ , we receive the implicit value for the interest rate, which corresponds to an equilibrium on the foreign exchange market for a fixed exchange rate regime:

$$E(s_{t+1}) = 0$$

$$r^* = r^{f*} + rp$$

It is necessary to realize that the equilibrium exchange rates for the accession countries are not constant in time and that the movements of the actual exchange rate values depend on many economic and institutional factors. That implies that, for the accession countries, the interest rate determined by the real interest rate parity can differ from the neutral interest rate, particularly in the fixed exchange rate regime, and this can represent a dilemma for the economic policy makers.

Bernhardtsen (2005) and Archibald and Hunter (2001) determine the equilibrium rates for small countries in this way. This approach requires an explicit estimate of the risk premium, both texts use expert estimates.

## 2.6 The long-term equilibrium interest rate

A concept similar to the neutral interest rate, but somewhat broader, is the long-term equilibrium interest rate. According to Bernhardtsen, it is the interest rate corresponding to the general equilibrium, the state where there are no forces for a redistribution of resources, all agents behave rationally, the environment is stable and the entire economy grows at a constant growth rate. An important difference compared to the neutral interest rate is that some partial markets can be in disequilibrium when determining the neutral interest rate, or a changing environment is allowed (for example a change in the tax system). This concept is very abstract and sometimes it is defined by means of intertemporal optimization models (the Ramsey model). Such an equilibrium interest rate then depends on parameters such as technological progress, intertemporal substitution elasticity and the subjective discount rate of consumers.



Ramsey (1926) has formulated a well-known model, which includes the firms and household sectors at a never-ending time horizon. The aim of the model was to determine the optimum rate of saving, which maximizes the total utility function of households, which is a function of the consumption in the individual periods. Similarly to the use of the common interest rate for the calculation of the present value of future cash flows, the subjective discount rate is used for the aggregation of immediate utility (the consumption function in a particular time period) in time to total utility over time. The higher it is, the more households prefer earlier consumption to later consumption. The elasticity of substitution is a parameter of the immediate utility function (for one time period), which indicates, how easy or difficult it is for the households to be able to replace consumption in one period by consumption in the subsequent period or to which degree they are willing to bear fluctuations in the consumption in the individual time periods. If  $\theta = 0$ , the households do not insist on an evenly distributed consumption, the higher  $\theta$  is, the more evenly distributed consumption is required by the households.

The model assumes a substitution production function with a decreasing marginal product of capital and work, constant returns to scale in intensity form<sup>6</sup> and labor-saving technological progress. Consumption and saving are defined in the form of the given quantity's ratio to the number of effective workers. The system of differential equations finally yields a stationary solution for consumption and capital defined in this way, so that pre capita consumption and per capita capital grow at the constant rate  $g$ , which equals the growth of labor-saving technological progress. The optimum levels of per capita consumption  $c^*$  and per capita capital  $k^*$  are implied by the conditions derived from the stationary solution to the said system of differential equations, where  $f(k)$  being the production function in intensity form and  $f'(k)$  its first derivative with respect to  $k$ ,  $n$  being the population growth rate):

$$f'(k^*) = \rho + \theta g \quad \text{and}$$

$$c^* = f(k^*) - (n + g)k^*$$

The first condition is related to the requirement that the marginal rate of substitution between consumption at time  $t$  and  $t+1$  equal the marginal rate of transformation between production at time  $t$  and  $t+1$  (the Ramsey-Keynes rule, see Blanchard and Fisher 1989 for details). The second condition is related to the balance of production between consumption and investment, with the need of sufficient capital caused by population growth and technological progress being taken into account.

The condition of profit maximization for firms implies that the marginal product of capital equals the interest rate (the model works with a net product after deduction of depreciation), so

that after putting in into the first equation we get the relation  $r = \rho + \theta g$ , so that the interest rate is an increasing function for all parameters and if  $\theta = 0$  or  $g = 0$ , it equals the subjective discount rate. If the per capita product grows and the households are not neutral against fluctuations in consumption, the equilibrium interest rate determined in this way is higher than the subjective discount rate.

An equation similar to the Ramsey-Keynes rule can be derived even without the full model, just by optimizing the objective function, mathematically implemented by the co-called Euler equation (an outline of its derivation can be found in the Nakagawa-Oshima 2000 study). In this case, the parameter  $g$  corresponds to the rate of consumption growth. This model is more general, because it does not assume a harmonic growth.

There are several estimates of parameters of intertemporal utility functions (e.g. Mankiw), which assume constant parameters. Although it can be assumed in practice that the rate of technological progress changes in time, overall this implies a low variability of equilibrium interest rates defined in this way. The above mentioned relation for the interest rate can be integrated in the Kalman filter, like Laubach and Williams (2003) have done it. In this paper we present a model with an alternative definition of disequilibrium components without the Philips curve.

### 2.7 The equilibrium interest rate determined by the Golden Rule

In this context, the Golden Rule (do to others what you want them to do to you) relates to keeping the level of capital endowment of effective labor constant. If the population grows or technological progress saves labor, it is necessary to provide additional capital for the effective labor force increment, so that the capital endowment of (effective) workers remains constant. The basis for the application of the Golden Rule is the second condition for optimum Ramsey model levels:

$$c^* = f(k^*) - (n + g)k^*$$

By taking a derivative with respect to  $k^*$  we get:

$$0 = f'(k^*) - (n + g) \quad \text{alebo} \quad r = f'(k^*) = n + g$$

This condition determines the optimum capital endowment of workers  $k^*$ , which maximizes consumption in a state of equilibrium defined by the Solow model. Let us assume a situation where the households save a constant portion of the production (determined by the parameter  $s$ , the propensity to save) under a non-specified value of capital endowment  $k$ . It then holds that:

$$sf(k) = dk/dt + (n + g)k,$$

where  $dk/dt$  ist the change in the capital endowment.

<sup>6</sup> The assumption of constant returns to scale implies that labor productivity can be expressed as  $f(k) = F(K, L)/L = F(k, 1)$ , where  $k = K/L$  is the capital/labor endowment.



<sup>7</sup> The particular form of the rule depends on the assumptions of the model used, so that various versions can be found in the literature, sometimes even the relation  $r = f'(k^*) = \rho + \theta g$

If  $k$  is lower than  $k^*$ , the saving (the left-hand side of the equation) is higher than the investment preserving a constant endowment of effective workers (the right-hand side of the equation), because the value of  $k$  will increase. If, on the other hand,  $k$  is greater than  $k^*$ , saving will not be sufficient to cover the growth of effective labor force and the value of  $k$  will fall down to  $k^*$  ( $dk^*/dt = 0$  from the definition). In this sense, the Golden Rule complies with Wicksell's requirement, because it corresponds to the long-term equilibrium between investment and saving.

The Solow model maximizes total consumption irrespective of the time preferences of households. If the households prefer consumption in the present to consumption in the future,

it can be derived (Blanchard and Fisher, 1989) that the equilibrium interest rate also depends on the subjective discount rate. This condition is called the Modified Golden Rule<sup>7</sup>. If this rule is derived by extending the model for the Golden Rule, the Modified Golden Rule leads to a higher value of the equilibrium interest rate and to a lower value of the equilibrium capital endowment and consumption as the original Golden Rule.

The determination of the equilibrium interest rate by means of the Golden Rule consists of the choice of the version of the appropriate condition and a calibration of the parameters. The estimates consist in defining the appropriate condition and calibration of parameters.

*(To be continued in the next number)*