APPLICATION OF GARCH MODELS IN FORECASTING THE VOLATILITY OF THE SLOVAK SHARE INDEX (SAX)

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The development of econometrics led to the invention of adaptive methods for modelling the mean value of the variable in question, the most widely used of which are the ARIMA methods (Box and Jenkins, 1970) and methods derived from them. The GARCH (Generalised Autoregressive Conditional Heteroskedasticity) method is one of the techniques based on the assumption that the random component of the model shows changes in variability. It was developed in a simplified form by Engle (1982) and later generalised by Bollerslev (1986). The model was applied successfully in modelling the changing variability (or volatility) of the variable in time series, with the applications being taken in large measure from the area of financial investments. After identifying an asymmetric relationship between conditional volatility and conditional mean value, the econometrists focused their efforts on the design of methods for the modelling of this phenomenon.

Nelson (1991) proposed an exponential GARCH (EGARCH) model, based on a logarithmic expression of the conditional variability in the variable under analysis. Later, a number of modifications were derived form this method. One of them is the TARCH method (Threshold ARCH), which was introduced by Zakoian (1994). Practical experience in this area was described by Bollerslev, Chou and Kroner in full detail (1992). The application of the GARCH model in the conditions of the Czech capital market was studied by Hančlová (2000).

The purpose of this article is to quantify the aforementioned three models, using the values of the Slovak Share Index (SAX) from the period 1 August 1997 to 27 September 2002, representing 1,173 observations. The first 1,000 values were used for the quantification and statistical verification of the model, and the last 173 for the demonstration of a forecast ex post.

Fig. 1 The first thousand values of the SAX index observed

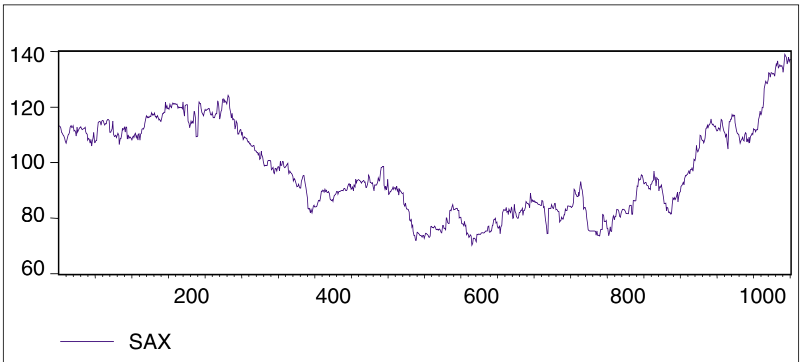
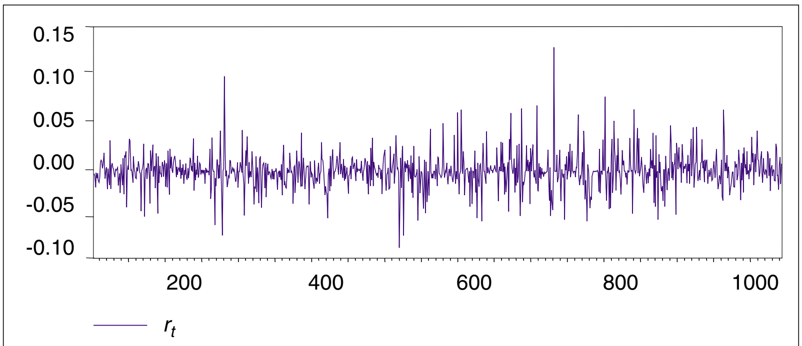


Fig. 2 The first thousand observations of the returns of SAX time series.



SAX index returns follow a martingale

If the value of the stock market index at time t is marked P_t, the return of the index at time t is given by the following equation:

$$r_t = ln(P_t / P_{t-1}).$$

We assume that stock market index returns follow a martingale process, i.e. they can be modelled with the help of the following equation:

$$r_t = \mu + e_t, \qquad (1)$$

where μ is the mean value of the return, which is expected to be zero; e_t is a random component of the model, not autocorrelated in time, with a zero mean value.[2] Sequence e_t et may be considered a stochastic process, expressed as:

$$e_t = z_t + \delta_t,$$

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[2]A more frequently used form of this equation is: $ln(P_t) = ln(P_{t-1}) + e_t$

where z_t is a stochastic variable not autocorrelated in time, with a standardised normal distribution. δ_t is the conditional variance of returns at time t, the changes of which will be modelled by means of the presented models.

The GARCH model

The GARCH method has a wide range of capital markets applications. The model is based on the assumption that forecasts of variance changing in time depend on the lagged variance of capital assets. An unexpected increase or fall in the returns of an asset at time *t* will generate an increase in the variability expected in the period to come.

A general GARCH (*p, q*) model is given by the following equation:

$$\sigma_t^2 = a + \sum_{i=1}^{q} b_i e_{t-i}^2 + \sum_{j=1}^{p} c_j \sigma_{t-j}^2 + w_t, \qquad (2)$$

where *p* is the degree of GARCH; *q* is the degree of the ARCH process; and w_t a random component with the properties of white noise. Since the equation expresses the dependence of the variability of returns in the current period on data (i.e. the values of the variables e_{t-i}^2 a δ_{t-j}^2) from previous periods, we denote this variability as conditional.

In determining the degrees *p, q* (i.e. identifying the model), we make use of the fact that the identification of GARCH is based, from the methodological point of view, on the same principles as the ARMA method (see Box-Jenkins, 1970), while the degrees *p, q* are identified by means of the autocorrelation function and a partial autocorrelation function of the square of residues. The basic and most widespread model is GARCH (1,1), which can be expressed as:

$$\sigma_t^2 = a + b e_{t-1}^2 + c \sigma_{t-1}^2 + w_t. \qquad (3)$$

As the variance is expected to be positive, we expect that the regression coefficients *a, b, c* are always positive, while the stationarity of the variance is preserved, if the coefficients *b* and *c* are smaller than 1.0.

The conditional variability of the returns defined in (3) is determined by three effects:

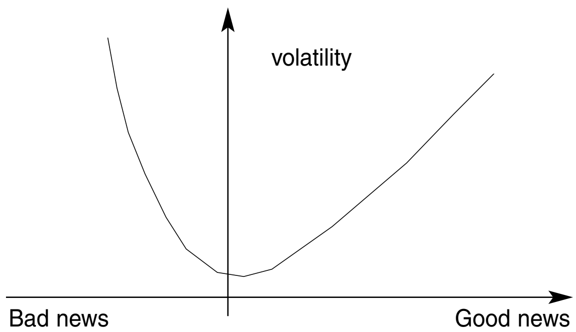
1. the constant part, which is given by the coefficient *a*;
2. the part of variance expressed by the relationship $b e_{t-1}^2$ and designated as ARCH component;
3. the part given by the predicted variability from the previous period and expressed by the relationship $c\delta_{t-1}^2$. This component is termed as GARCH.

The sum of regression coefficients *(b + c)* expresses the influence of the variability of variables from the previous period on the current value of the variability. This value is usually close to 1.0, which is a sign of increased inertia in the effects of shocks on the variability of returns on financial assets.

Asymmetric effect

The principal disadvantage of the GARCH model is its unsuitability for modelling the frequently observed asymmetric effect, when a different volatility is recorded systematically in the case of good and bad news. In the case of martingale models, falls and increases in the returns can be interpreted as good and bad news. If a fall in returns is accompanied by an increase in volatility greater than the volatility induced by an increase in returns, we may speak of a 'leverage effect'. This idea is illustrated in Fig. 3. Suitable instruments for the modelling of an asymmetric affect are the TARCH and EGARCH models.

Fig. 3 Leverage effect – the reaction of volatility to good and bad news



The TARCH model

The TARCH model is an asymmetric model. Its basic variant is TARCH(1,1), which is expressed by an equation for the modelling of a conditional variance:

$$\sigma_t^2 = a + b e_{t-1}^2 + c \sigma_{t-1}^2 + d e_{t-1}^2 \xi_{t-1} + w_t, \qquad (4)$$

where
a) $\xi_{t-1} = 1$, if $e_{t-1} < 0$,
b) $\xi_{t-1} = 0$, if $e_{t-1} > 0$.

The model is based on the assumption that unexpected (unforeseen) changes in the returns of the index r_t expressed in terms of e_t, have different effects on the conditional variance of stock market index returns. An unforeseen increase is presented as good news and contributes to the variance in the model through multiplicator *b*. An unforeseen fall, which is a piece of bad news, generates an increase in volatility through multiplicator *b+d*. The asymmetric nature of the returns is then given by the non-zero value of the coefficient *d*, while a positive value of *d* indicates a 'leverage effect'.

The EGARCH model

In this model, the conditional variance may be expressed as follows:

$$\ln(\sigma_t^2) = a + b\ln(\sigma_{t-1}^2) + c\,\frac{e_{t-1}}{\sigma_{t-1}} + d\left|\frac{e_{t-1}}{\sigma_{t-1}}\right| + w_t. \quad (5)$$

The form of the equation indicates that conditional variance is an exponential function of the variables under analysis, which automatically ensures its positive character. The exponential nature of EGARCH ensures that external unexpected shocks will have a stronger influence on the predicted volatility than TARCH. An asymmetric effect is indicated by the non-zero value of *c* and the presence of a 'leverage effect' is shown by its negative value.

Test for a martingale process

The results obtained from the quantification of equation (1) are shown in Table 1.

Tab. 1 Results of regression modelling the martingale nature of the mean value of SAX index returns

Statistics	
μ	0.000193
Durbin – Watson	2.017521
Coefficient of determination	0.000000
Sum of squares of residuals	0.255457
LAG of statistically significant autocorrelation	
Ljung – Box ***	–
Ljung – Box **	–
Ljung – Box *	–
LAG of statistically significant heteroskedasticity	
Ljung – Box ***	–
Ljung – Box **	–
Ljung – Box *	5
Normality	
Jarque – Berra	1342.749***

(* means ten, ** five, and *** one-percent level of significance)

The mean value of returns μ does not deviate significantly from zero in statistical terms, hence the coefficient of determination will also have a zero value. The absence of autocorrelation in random components e_t is indicated by the value of Durbin-Watson statistics, which is close to 2.0 (an insignificant value). Autocorrelation was not even shown by the individual members of the estimated autocorrelation function given by Ljung-Box statistics, with none of them being statistically significant. On the contrary, in a test for the presence of heteroskedasticity of residuals, where we used an estimate of an autocorrelation function of the square of residuals, a statistically significant shift of 5 periods was recorded, which allows us to speak of the variability shown by random components. At a one-percent level of significance, the estimated model showed a deviation from the normal distribution of random components e_t.

Tab. 2 Quantification and statistical verification of GARCH, TARCH, and EGARCH models

Statistics	GARCH(1,1)	EGARCH(1,1)	TARCH(1,1)
μ	0.000395	-0.000161	-6.20E-05
a	2.87E-05***	-1.119736***	4.60E-05***
b	0.089354***	0.874512***	-0.021041**
c	0.805020***	-0.109250***	0.758276***
d	–	0.110361***	0.182597***

(* means ten, ** five, and *** one- percent level of significance)

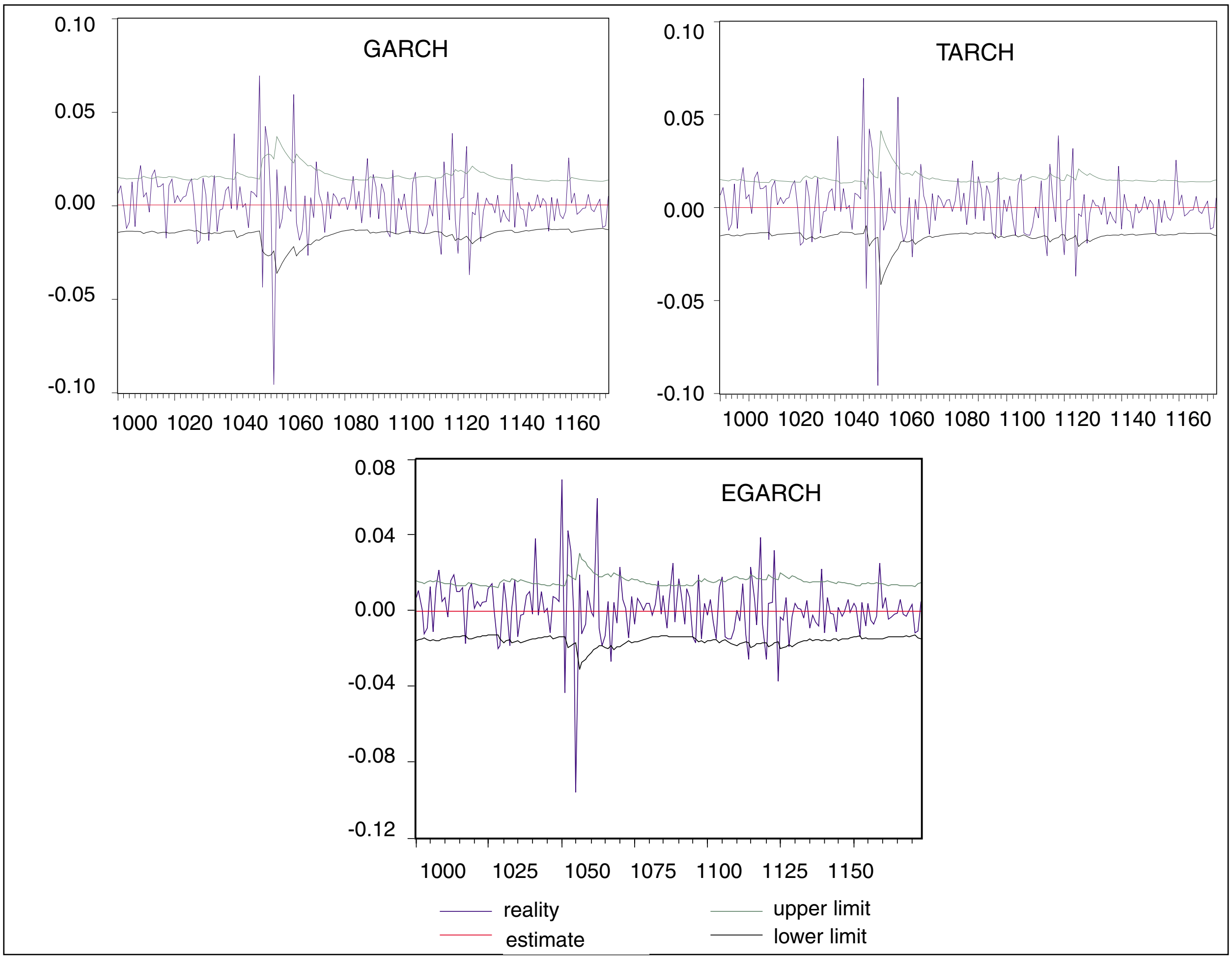
Quantification and verification of models of a conditional variance

The results of estimation and statistical verification of GARCH(1,1), TARCH(1,1), and EGARCH(1,1) models are shown in Table 2. The results indicate that the GARCH components of the variance are statistically significant in all three models. In the case of the regression coefficient *b* in the TARCH model, this coefficient has a negative value, which may lead to a negative conditional variance at certain values of the variables in equation (4). The sum of the coefficients *(b + d)* in the case of GARCH is close to 1.0, which is a sign of inertia in the development of the conditional variance. The existence of a 'leverage effect' was confirmed in the case of both asymmetric models. This indicates that, of the properties analysed above, the best results are achieved with the EGARCH model, which, unlike GARCH models asymmetric effects, without being exposed to the danger of having to predict a negative variance.

Forecasting the volatility of the SAX index ex post

The last 173 observations in the time series of SAX were used for an ex-post forecast, with the main focus on the forecast of volatility. The graphs for the period under analysis indicate that the interval estimate of returns made at a level of reliability of 67 percent is not constant in the case of all the three models and takes into account the changing variance of the variable in question. This means that, unlike classical approaches based on the

Fig. 4 Ex-post forecast of the volatility of SAX index returns



assumption of a constant variance of random components, the EGARCH, GARCH, and TARCH models react to the actual changes in the volatility of the returns.

Conclusion

In conclusion, we may say that the Slovak Share Index (SAX) follows a martingale process. The variability of its conditional variance can be modelled by means of TARCH, GARCH, and EGARCH models. The best results were achieved with the EGARCH model. In the period under review, the existence of an asymmetric effect (which was connected with a 'leverage effect') was confirmed. In future, these results can be extended to include those of other variants of the GARCH model, and the proposed methodology may be used for the evaluation of option prices as well.

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