Inflationary consequences of the indebtedness levels of the member countries in a monetary union

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Abstract: This paper studies the impact on the common monetary policy and the externalities due to the various levels of indebtedness of the member countries in a monetary union. While the common central bank tends to conduct a more accommodative monetary policy in order to avoid the default of the moderately indebted member countries, the most heavily indebted countries have to default. More precisely, the optimal inflation rate increases as the weight given by the common central bank to the aim of price stability gets smaller, as the number of highly indebted countries in the union grows, and as the share of nominal debt of these countries in the global indebtedness of the monetary union gets higher. Besides, the optimal inflation rate increases as the interest rate on the risk free capital gets higher and as the interest rate on inflation indexed bonds in the fiscally weak countries is reduced. Finally, the optimal inflation rate increases as the incentive effect on the issuing of public debt of a smaller cost of default decreases, and then it is an increasing function of this cost of default in the fiscally weak countries.

Keywords: monetary policy, monetary union, public debt, inflation rate

JEL Classification Codes: E63, H63, H77, H87

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I. Introduction

The member countries of a monetary union lose their autonomy to conduct independent monetary and exchange rate policies. And their budgetary policies can neither remain widely independent and autonomous. Indeed, the common monetary policy affects the fiscal policies, the budgetary deficits and the public debts, as well as these fiscal policies themselves influence the behavior of the common central bank and contribute to direct and to define the common monetary policy. The Fiscal Theory of the Price Level has particularly contributed to show the impact of the budgetary policy and of the level of indebtedness on the inflation rate and on the optimal monetary policy. Moreover, in Europe, the monetary unification has reinforced the narrowness and the complexity of these monetary and fiscal interdependencies; it has modified the public debt management as well as its consequences on the inflation rate.

First, the creation of the Economic and Monetary Union (EMU) has changed public debt management (Favero et al., 2000). Indeed, the speculative demand and the demand for portfolio diversification related to exchange rate variations have disappeared, as all the bonds are now denominated in Euros. Today, the competition between EMU member States in issuing government securities only concerns the liquidity and the risk of default (credit risk) of their assets. Some countries may be tempted to adopt non cooperating behavior in the choice of the issue dates or in the information about the quality of their assets. However, debt structures and maturities (debt duration) have strongly converged in EMU, and an efficient and liquid market for debt instruments has been created, which contributes to ease the conduct of the common monetary policy: the liquidity and the transmission channels of monetary policy are more easily controllable. This market relies essentially on fixed-rate medium and long term bonds, exchanged on well-integrated markets with large trade volumes, whereas the share of the markets for variable rates securities has much been reduced (Favero et al., 2000; de Haan and Wolswijk, 2005).

The longer term duration of debt implies that the investors support the anti-inflationary policy of the European Central Bank (ECB), as well as it is explained by the greater price-stability provided by an independent central bank. Moreover, it also contributes to isolate the governments’ budget from monetary policy and variations in interest rates. Indeed, in the private sector, minimizing interest rates costs or the risks of large fluctuations in these payments are the key considerations for debt management. However, in the public sector and at the level of the European Union (EU), macroeconomic goals also tend to be important. They can include the macroeconomic stabilization (smoothing tax rates, stabilizing public deficits, etc), the development of national financial markets, or the support of monetary policy (de Haan and Wolswijk, 2005). In this context, Missale (2001) studies the optimal debt management in the framework of the Stability and Growth Pact, introducing deficit stabilization as a new important objective. Then, he finds that a longer maturity structure of conventional debt is optimal if the ECB places a lower weight on output stabilization than national monetary authorities and if the EMU member States are hit by asymmetrical shocks. Besides, the lower the weight that the ECB assigns to output stabilization, the more attractive is inflation indexed debt for deficit stabilization.

In these conditions, how can this more integrated public debt market impact the common monetary policy, and the optimal inflation rate in a monetary union?
In a discretionary monetary policy setting, giving rise to inflation bias, Beetsma and Bovenberg (1999) show that monetary unification relaxes budgetary discipline and increases public debt accumulation. This harms welfare, if the governments are sufficiently myopic and give a higher preference than the society to the current economic situation. Debt ceilings should then usefully be introduced in a monetary union, in order to allow an independent central bank to preserve price stability. In the same way, Traficante (2009) shows that in a monetary union, the inter-temporal solvency constraint for the governments only holds at the aggregate level. Thus, without ceilings on the debt levels, a country could become the permanent debtor of its partners. The coordination between the budgetary policies of the member countries is thus necessary. The coordination between the monetary and budgetary authorities is also necessary, according to Van Aarle et al. (1995). Indeed, the authors underline the strategic interaction between the monetary authority which controls monetization and the fiscal authorities which control primary fiscal deficits. A conflict can therefore arise if these authorities have distinct objectives regarding inflation, debt stabilization or public spending. Nevertheless, Van Aarle et al. (1997) also show that in a monetary union, the central bank can’t really be exploited as common property by undisciplined governments: in case of fiscal difficulties, they can’t rely on a more accommodative monetary policy. Debt stabilization is quicker with a common central bank than with individual national central banks, and inflation as well as fiscal deficits are then lower. Indeed, the strategic position of the common central bank strengthens, as governments lose power due to their smaller relative economic size.

Regarding the consequences of the public debt levels on the inflation rates in a monetary union, Bergin (2000) applies the Fiscal Theory of Price Level determination to the case of a monetary union. He finds that a rise in the debt level by one member government can raise the common price level, suggesting a role for fiscal solvency rules in a monetary union. In the same way, in a closed economy setting, Leith and Wren-Lewis (2000) assume that when monetary policy seeks to raise interest rate in case of inflationary tensions, a self stabilizing and contractionary budgetary policy is necessary to ensure the long run stability of the model. On the contrary, a fiscal policy that doesn’t, by itself, ensure fiscal solvency constrains the monetary policy to be more ‘passive’ and to accept a possible higher level of inflation by decreasing real interest rates. Woodford (1996) also shows that debt limits are a necessary condition to be able to charge the common central bank of a monetary union with responsibility for maintaining a stable value for the common currency. Indeed, an independent central bank conducting a steady non inflationary monetary policy is not sufficient to achieve price stability. A country that shares a common currency with another one exposes itself to price level instability and to the fluctuations in economic activity resulting from the fiscal instability in the other country, even if it is itself a model of fiscal probity.

Besides, Beetsma and Vermeylen (2007) study the implications of monetary unification for real interest rates and relative debt levels. They find that the common currency makes the inflation rates and the risk return characteristics of the participating countries more similar, so that the substitutability of their public debts increases after monetary unification. Then, the share of the debt issued by undisciplined governments increases, as well as the average expected return on the debt. The relative debt levels may thus become a source of tensions for the political sustainability of the monetary union. Nevertheless, Creel and Le Cacheux (2007) contest the hypothesis that the divergences in the inflation dynamics would tend to vanish in a monetary union. They assume that debt levels and real returns on these debts are not fully substitutable in a monetary union, as they contribute to finance more or less productive investments. Therefore, according to them, imposing homogeneous fiscal rules, like those of the Stability and Growth Pact, to the heterogeneous members of a monetary union would be necessary to ensure the same degree of price level stability as in a monetary union.
union may be counterproductive. Finally, Jahjah (2000) shows that in a monetary union with an independent central bank and a sufficiently large number of relatively small member countries, the latter tend to accumulate less debt, and thus, that an equilibrium with no inflation and no default on the debt of the governments exists. However, a highly indebted country would be more likely to default if it joins the monetary union than if it remains outside. Therefore, the monetary unification would have substituted a default risk to the inflationary risk, in case of high levels of indebtedness.

In the same tradition, this paper aims at studying the consequences of the various indebtedness levels of the member countries of a monetary union on the optimal inflation rate and on the monetary policy of the common central bank. We want to shed light on the transmission channels of the inflationary tensions according to the various budgetary policies and indebtedness levels of the member countries of a monetary union. Indeed, in case of high debt levels of some member countries, the common central bank has two main options. It can inflate away this high debt levels and create inflationary tensions in all the monetary union; or it can let the country with fiscal difficulties default on part of its debt. In this framework, our paper shows that the optimal inflation rate increases as the weight given by the common central bank to the aim of price stability gets smaller, as the number of highly indebted countries in the union grows, and as the share of nominal debt of these countries in the global indebtedness of the monetary union gets higher. Besides, the optimal inflation rate increases as the interest rate on the risk free capital gets higher and as the interest rate on inflation indexed bonds in the fiscally weak countries is reduced. Finally, the optimal inflation rate increases as the incentive effect on the issuing of public debt of a smaller cost of default decreases, and then it is an increasing function of this cost of default in the fiscally weak countries.

The structure of the paper is as follows. The second section describes the model and the representative economic agents. The third section defines, according to the levels of indebtedness of three member countries types in the monetary union, the optimal monetary policy and inflation rate. The fourth section analyzes the empirical implications of our model for the public debt management and the inflation rate in the EMU. Finally, the fifth section concludes the paper.

**II. The model**

We consider a monetary union of (n) member countries. In each country, a benevolent government maximizes the utility of a representative agent. This government accumulates a public debt in period 1, and must fully pay back in period 2 this public debt. To repay the debt, according to the monetary policy, the government has the choice between raising taxes, decreasing public expenditures, or defaulting, whose cost is proportional to the amount which is defaulted. The government maximizes a function of welfare for the representative citizen depending on the inflation level, on the economic activity, and on the risk of default. The budgetary policies are decentralized and defined at the national level.

Furthermore, there is a common central bank, which defines the common monetary policy for all the monetary union, and which is concerned about the average welfare in the entire monetary union. Its aim is to achieve price stability, but also to ensure the stability of all the financial system. Without barriers to commodity trade, the inflation rate is supposed to be identical in all the monetary union. Let’s mention that contrary to Jahjah (2000) for example, we don’t consider here the revenues that could be derived from seigniorage. Indeed,
they are empirically relatively small in Europe today. Moreover, we want to study the implications of monetary unification on the public debt levels, on the risks of default and on the inflation rate in the member countries, even in the absence of any seigniorage revenues.

We consider here the following sequential game. First, the optimal fiscal responses and strategies of the governments (tax rates, public debts, default) are defined, taken the inflation rate as given. Then, the central bank, acting as a Stackelberg leader, fixes the optimal inflation rate for all the monetary union, given these fiscal reaction functions (see the following section II.3. for the justification for the timing of this sequential game).

1. The representative agents

There are two periods in our model. In period 1, for example before the advent of the monetary union, the government of the country \( i \) issues nominal bonds whose interests are \( b_i \) for an amount \( B_{i,1} \), and inflation indexed bonds whose interests are \( z_i \) for an amount \( Z_{i,1} \). This distinction between nominal and inflation indexed bonds is the main divergence and the main contribution of our paper in comparison with Jahjah (2000), for example. The individuals buy these bonds; they also invest in capital \( I_{i,1} \) whose real interest rate is \( r \). As capital is highly mobile at the international level, this interest rate \( r \) is supposed to be identical in all member countries of the monetary union. We also make the hypothesis that the individuals are perfectly rational, and thus, are indifferent between holding capital or bonds, whose returns must therefore be equalized:

\[
(1 - d_i)(1 + b_i - \pi) = (1 - d_i)(1 + z_i) = (1 + r)
\]

Where \( \pi \) is the inflation rate in the monetary union, and \( d_i \) is the probability of default of the country \( i \) on its debt in period 2.

The real return of the nominal bonds adjusted for the inflation rate and for the probability of default, equals the real return of the inflation indexed bonds adjusted for the probability of default, and equals the real return of the risk free capital.

In period 1, the saving of the agents is the share of their resources (labor revenues net of public taxes) which is not consumed. This saving can be used to buy capital or to buy bonds of the public debt.

\[
S_{i,1} = (1 - t_{i,1}) Y_{i,1} - C_{i,1} = I_{i,1} + B_{i,1} + Z_{i,1}
\]

Where \( S_{i,1} \) is saving and \( C_{i,1} \) is consumption of the representative agent; \( t_{i,1} \) is the taxation rate; \( Y_{i,1} \) are real labor revenues, in country \( i \) in period \( t \).

In period 2, individuals consume all their wealth: labor revenues net of public taxes, as well as the returns on capital and on public bonds. Therefore, with (1) and (2), we have:

\[
C_{i,2} = (1 - t_{i,2}) Y_{i,2} + (1 + r) I_{i,1} + (1 - d_i)(1 + b_i - \pi) B_{i,1} + (1 - d_i)(1 + z_i) Z_{i,1}
\]

\[
= (1 - t_{i,2}) Y_{i,2} + (1 + r) (1 - t_{i,1}) Y_{i,1} - (1 + r) C_{i,1}
\]

We can make the hypothesis that for the individuals, there is a discount factor \( \rho > 0 \) which measures their preference for the present. If this parameter is small, the representative agent is risk neutral and has no preference for the present. On the contrary, the higher is this parameter, the higher is the preference for the present, and the higher is the risk-aversion of the representative agent. Thus, we suppose that the inter-temporal utility of the representative agent, which must be maximized, has the following expression:

\[1\]

\[\text{We can suppose that the economic activity in the second period (}Y_{i,2}\text{) is not exogenous, but depends on this previous investment in capital (}I_{i,1}\text{).} \]
\[ U_i = C_{t,i} + \frac{1}{(1 + \rho)} C_{t,2} \] (4)

2. The governments

In period 1, for example before the creation of a monetary union, the governments chose their initial debt levels as well as their debt portfolios. We suppose that the debt of the country (i) in relation to the global indebtedness of the monetary union is an increasing function of the relative size (\( \gamma_i \)) and a decreasing function of the cost of default (\( c_i \)) for this country in the monetary union. Indeed, the country (i) can default on a share (\( d_i \)) of its debt; however, the default on one unit of public debt implies a cost (\( 0 < c_i < 1 \)), for example in terms of loss of reputation on the financial markets, which could imply higher future risk premiums. These costs are supposed to be heterogeneous across member countries of the monetary union. More precisely, if the cost of default is smaller in the country (i) than the average cost in the monetary union (\( c_i < c \)), the country (i) tends to issue more public debt, whereas it issues smaller amounts of debt if it bears higher costs (\( c_i > c \)). So, we obtain the following equation for the relative debt level of the country (i):

\[
\frac{(B_{i,1} + Z_{i,1})}{(B_1 + Z_1)} = \gamma_i - \lambda(c_i - c) \quad \text{with: } \lambda > 0 \quad c = \frac{1}{n} \sum_{i=1}^{n} c_i
\] (5)

where (\( \gamma_i \)) is the relative weight, for example in terms of relative GNP, of the country (i) in the monetary union: this parameter can thus be interpreted as a measure of the relative size of the country (i) in the monetary union.

We will discuss the choice of the debt portfolio in the section III.1. Furthermore, in period 1, the budgetary constraint of the government (i) implies:

\[ G_{i,1} - t_{1,i} Y_{i,1} = B_{i,1} + Z_{i,1} > 0 \] (6)

Where (\( G_{i,1} \)) are the real public expenditures of the government (i) in period t.

In period 2, after the creation of a monetary union, the government (i) must pay back the public debt contracted in the previous period. Then, it can default on a share (\( d_i \)) of this debt; however, we have mentioned that the default on one unit of public debt implies a cost (\( 0 < c_i < 1 \)). Therefore, in period 2, the budgetary constraint of the government (i) is:

\[ t_{i,2} Y_{i,2} - G_{i,2} - c_i d_i (1 + b_i - \pi) B_{i,1} - c_i d_i (1 + z_i) Z_{i,1} = (1 - d_i) (1 + b_i - \pi) B_{i,1} + (1 - d_i) (1 + z_i) Z_{i,1} > 0 \] (7)

Indeed, the resources of the governments (taxes) are equal to its expenditures: public expenditures, and repayment of the previously contracted public debt, net of the inflation reducing the service of nominal bonds, and net of the probability of default. As formerly mentioned, output in period 2, and therefore also the fiscal resources, are reduced in proportion of the amount of the public debt defaulted by the government.

Combining the former equations (1) to (7), we have:

\[ U_i = (Y_{i,1} - G_{i,1}) + \frac{1}{(1 + \rho)} (Y_{i,2} - G_{i,2}) + \frac{(r - \rho)}{(1 + \rho)} l_{i,1} - \frac{c_i d_i (1 + z_i) (\gamma_i - \lambda c_i + \lambda c)}{(1 + \rho)} (B_1 + Z_1) \] (8)

\[ U_i = (1 - t_{i,1}) Y_{i,1} + \frac{1 - t_{i,2}}{(1 + \rho)} Y_{i,2} + \frac{(r - \rho)}{(1 + \rho)} l_{i,1} + \frac{(z_i - d_i - z_i d_i - \rho) (\gamma_i - \lambda c_i + \lambda c)}{(1 + \rho)} (B_1 + Z_1) \] (9)

\[ U_i = (Y_{i,1} - G_{i,1}) + \frac{[(1 - c_i + c_i t_{i,2}) Y_{i,2} - G_{i,2}]}{(1 + \rho) (1 - c_i)} + \frac{(r - \rho)}{(1 + \rho)} l_{i,1} - \frac{c_i (1 + z_i) (\gamma_i - \lambda c_i + \lambda c) (B_1 + Z_1)}{(1 + \rho) (1 - c_i)} \] (10)
Therefore, in the first period, the benevolent government of the country \((i)\) can choose the public expenditures \((G_{i,1})\) such as \(\partial U_i/\partial G_{i,1}=0\) in equations (9) or (11), in particular according to the fiscal multiplier’s value in the country. Or it choose the taxation rate \((t_{i,1})\) such as \(\partial U_i/\partial t_{i,1}=0\) in equation (10), according to the disincentive effect of high taxation rates on its economic activity. However, the government of the country \((i)\) can’t choose simultaneously both parameters. Afterwards, in the second period, this government has three possibilities in order to maximize the inter-temporal utility of the representative agent, taking the inflation rate \((\pi)\) and its own choices during the first period as given. It can set the taxation rate \((t_{i,2})\) and the share of defaulted debt \((d_i)\) such as \(\partial U_i/\partial t_{i,2}=(\partial U_i/\partial d_i)=0\) in equation (9), which would then imply a given level of public expenditures. Or it can set the public expenditures \((G_{i,2})\) and the share of defaulted debt \((d_i)\) such as \(\partial U_i/\partial G_{i,2}=(\partial U_i/\partial d_i)=0\) in equation (8), which would then imply a given taxation rate. Or it can set the public expenditures \((G_{i,2})\) and the taxation rate \((t_{i,2})\) such as \(\partial U_i/\partial t_{i,2}=(\partial U_i/\partial G_{i,2})=0\) in equation (10), and the share of defaulted debt would then be the consequence of these fiscal policies.

3. The share of defaulted debt

Let’s consider our last hypothesis, where the government \((i)\) chooses the optimal fiscal policies (taxation rates and public expenditures) for the members of the country \((i)\). According to equation (10), these optimal policies \((t_{i,2}^{*} \text{ and } G_{i,2}^{*})\) and such as:

\[
\frac{\partial Y_{i,2}}{\partial G_{i,2}} = \frac{1}{(1-c_i + c_i t_{i,2})} \quad \text{and} \quad \frac{\partial G_{i,2}}{\partial t_{i,2}} = c_i Y_{i,2} \quad (11)
\]

Then, according to equation (7), the share of the defaulted debt by this government \((i)\) in the second period is:

\[
d_i = \frac{G_{i,2}^{*} + (1 + b_i - \pi)B_{i,1} + (1 + z_i)Z_{i,1} - t_{i,2}^{*}Y_{i,2}}{(1-c_i)[(1 + b_i - \pi)B_{i,1} + (1 + z_i)Z_{i,1}]} \quad (12)
\]

For the country \((i)\), the share of defaulted debt is therefore:

- \(d_i^{*}=1\) if: \(t_{i,2}^{*}Y_{i,2} \leq G_{i,2}^{*} + c_i(1 + b_i - \pi)B_{i,1} + c_i(1 + z_i)Z_{i,1}\)
  
  that is to say if: \(\pi \leq \pi_i\) such as:
  
  \[
  \pi_i = 1 + b_i + \frac{(1+z_i)Z_{i,1}}{B_{i,1}} - \left(\frac{t_{i,2}^{*}Y_{i,2}-G_{i,2}^{*}}{c_iB_{i,1}}\right) \quad (13)
  \]

  Indeed, if the fiscal resources are not enough to cover at least the cost of default on the debt, the country \((i)\) is sure to be insolvent even if it defaults on all its debt.

- \(d_i^{*}=0\) if: \(G_{i,2}^{*} + c_i(1 + b_i - \pi)B_{i,1} + c_i(1 + z_i)Z_{i,1} \leq t_{i,2}^{*}Y_{i,2} \leq G_{i,2}^{*} + (1 + b_i - \pi)B_{i,1} + (1 + z_i)Z_{i,1}\)
  
  that is to say if the fiscal resources are moderate, if: \(\pi \leq \pi \leq \pi_i\), then: \(0 \leq d_i^{*} \leq 1\) has the expression mentioned in (12). The government of the country \((i)\) defaults on part of its debt, and its budgetary constraint is verified and balanced in the second period.

- \(d_i^{*}=0\) if: \(t_{i,2}^{*}Y_{i,2} \geq G_{i,2}^{*} + (1 + b_i - \pi)B_{i,1} + (1 + z_i)Z_{i,1}\)
  
  that is to say if: \(\pi \geq \pi_i\) such as:
  
  \[
  \pi_i = 1 + b_i + \frac{(1+z_i)Z_{i,1}}{B_{i,1}} - \left(\frac{t_{i,2}^{*}Y_{i,2}-G_{i,2}^{*}}{B_{i,1}}\right) \quad (14)
  \]

If the fiscal resources are sufficient to pay the public expenditures and to reimburse the debt, the country \((i)\) doesn’t default; there is even a fiscal surplus in the second period.

\[\frac{\partial^2(d_i^*)}{\partial \pi^2} = \frac{-2b_i}{(1-c_i)[(1+b_i-\pi)B_{i,1} + (1+z_i)Z_{i,1}]} < 0.\]
So, if the inflation rate is too low, if monetary policy is too contractionary, servicing the public debt is so costly that the optimal choice for the government (i) is to default on all its debt; no equilibrium with a positive public debt can then exist. As the inflation rate increases, the share of the debt defaulted decreases, and finally, for a sufficiently high level of inflation, the government always chooses to repay all its debt without defaulting.

Furthermore, to allow the government (i) to meet its engagements, the necessary level of inflation is an increasing function of the stock of its inflation indexed debt ($Z_{i,1}$), of the interest rates on its public debt ($b_i$, $z_i$), of its public expenditures to be financed ($G_{i,2}$) and of the cost of default ($c_i$). On the contrary, it is a decreasing function of the taxation rate ($t_{i,2}$) and of the economic activity ($Y_{i,2}$) in the country (i).

4. The common central bank

The common central bank aims at minimizing the deviations of average inflation from a target ($\bar{\pi}$); in particular, price stability has been defined as the main aim of the European Central Bank. However, it also aims at preserving the stability of the financial system and at avoiding the risks of default from one government. Indeed, in the event of huge fiscal difficulties, the central bank can be forced to accommodate its monetary policy in order to avoid a fiscal crisis. Even if this risk can be null in good times and with fiscally strong governments, it cannot be avoided in trouble times and if some governments encounter large fiscal problems. Furthermore, we have seen that the recent fiscal difficulties of the Greek government to finance its public debt on the financial markets necessitate empirically, for the monetary authority, to conduct a more accommodative monetary policy. The integration of the risk of default in the loss function of the central bank is thus not in contradiction with its independence. We only want to take into account the necessity, for the central bank, to preserve the stability of the financial system by limiting the risks of default of the member countries of the monetary union. The non-standard loss function of the common central bank is therefore supposed to be as follows:

$$L^M = a^M (\pi - \bar{\pi})^2 + \sum_{i=1}^{\gamma} y_i d_i$$ (15)

Where ($\gamma_i$) has been previously defined as the relative size of the country (i) in the monetary union; ($a^M$) is the relative weight related to price stability in comparison with the stability of the financial system for the central bank; this parameter thus corresponds to the degree of conservativeness of the central bank.

Chari and Kehoe (2004) show that if the monetary authority can commit to follow a fixed monetary policy, there are no inflationary risks: fiscal constraints are thus useless and they can only involve costs. On the contrary, if the monetary authority cannot commit, then there is a time inconsistency problem because the central bank has an incentive to inflate away nominal debt; in these conditions, fiscal policy has a free-rider problem, and debt constraints may be desirable. Therefore, in order to have meaningful inflationary consequences of the fiscal policies, we make the hypothesis that the monetary authority takes into account the budgetary situation of the governments to fix the optimal inflation rate and that it is unable to commit. More precisely, we suppose a Stackelberg equilibrium where the monetary authority pre-commits to an inflation target ($\bar{\pi}$); however, the empirical effective inflation rate ($\pi$) is different and depends on the anticipated budgetary reaction function. Whatever the targeted level of inflation, the central bank would have interest in adapting its monetary policy to the fiscal behaviors and strategies of the governments. Therefore, the lack
of commitment in monetary policy is modeled by having the monetary policy choose the inflation rate after the choice of the debt contracts by the governments. In this framework, \( \frac{\partial L}{\partial \pi} = 0 \) implies the following inflation rate:

\[
\pi = \bar{\pi} - \frac{1}{2aM} \sum_{i=1}^{n} y_i \frac{\partial d_i}{\partial \pi} \geq \bar{\pi}
\]

(16)

In order to prevent the default from some governments, the optimal inflation rate fixed by the monetary authority must always be above its target.

### III. The optimal monetary policy

1. **Three types of countries**

Let’s suppose that there are 3 groups of countries in the monetary union: (A), (B) and (C), whose relative weights are such as: \( \gamma_A + \gamma_B + \gamma_C = 1 \).

- A group (C) of fiscally weak and very indebted countries, whose relative weight is \( \gamma_C \), and whose level of indebtedness is such as:

\[
(1 + b_i - \bar{\pi})B_{i,1} + (1 + z_i)Z_{i,1} \geq \frac{(t_{i,2}Y_{i,2} - G_{i,2}^*)}{c_i} \quad (\forall i \in C)
\]

(17)

- A group (B) of intermediary countries, whose relative weight is \( \gamma_B \), and whose level of indebtedness is such as:

\[
t_{i,2}^*Y_{i,2} - G_{i,2}^* < \left[ (1 + b_i - \bar{\pi})B_{i,1} + (1 + z_i)Z_{i,1} \right] \leq \frac{(t_{i,2}Y_{i,2}^* - G_{i,2}^*)}{c_i} \quad (\forall i \in B)
\]

(18)

Thus, these type (B) countries are liable to default, but only on a part of their debt.

- A group (A) of fiscally strong and very weakly indebted countries, whose relative weight is \( \gamma_A \), and whose level of indebtedness is such as:

\[
(1 + b_i - \bar{\pi})B_{i,1} + (1 + z_i)Z_{i,1} \leq t_{i,2}^*Y_{i,2} - G_{i,2}^* \quad (\forall i \in A)
\]

(19)

Thus, as \((-\pi < -\bar{\pi})\), these fiscally strong countries never default, and \( (d_A = 0) \).

If the situation of the monetary union is globally sound, that is to say if type (A) countries have the biggest weight, and if the fiscally weak type (C) countries are much less numerous \(^3\), the previous definitions also imply the following condition on the debt levels (see Appendix A): \( \frac{B_{C,1}}{Z_{C,1}} > \frac{B_{B,1}}{Z_{B,1}} > \frac{B_{A,1}}{Z_{A,1}} \). Therefore, the relative indebtedness of the governments in nominal bonds in comparison with their indebtedness in inflation indexed bonds is an increasing function of the indebtedness levels of the member countries. To the contrary, the issuing of inflation indexed bonds is an increasing function of the fiscal soundness of a country. So, the composition of the debt portfolio in indexed and non indexed bonds of the member countries of a monetary union depends endogenously on their debt levels.

In these conditions, Figure 1 represents the probability of default of types (A), (B) and (C) countries on their debt, according to the inflation rate in the monetary union.

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\(^3\) More precisely, the conditions mentioned in Appendix A are: \( (\gamma_A - \gamma_B)(B_{1} + Z_{1}) + (B_{B,1} + Z_{A,1}) - (B_{A,1} + Z_{A,1}) > 0 \) and: \( (\gamma_B - \gamma_C)(B_{1} + Z_{1}) + (B_{C,1} + Z_{C,1}) - (B_{A,1} + Z_{A,1}) > 0 \).
2. The optimal monetary policy

With a positive risk free real interest rate (r>0), no equilibrium exists where the investors would accept to lend to a country (i) which would be sure to default (d_i=1). Indeed, the interest rate on the nominal or inflation indexed bonds of this country would tend *ad infinitum*.

With an intermediary level of indebtedness, given the risk free interest rate (r), the yield on the bonds of the government (i) must be fixed at the level: 
\[ z_i = \frac{r+d_i}{1-d_i} \geq r \]
for the inflation indexed bonds, and 
\[ b_i = \frac{r+d_i}{1-d_i} + \pi \geq r + \pi \]
for the nominal bonds, in order to make the public debt as attractive as the risk free asset for the investors.

Finally, the real bond yields can be fixed at the levels (z_A=r; b_A=r+\pi), without risk of default, for the type (A) countries. So, the maximal level of debt that a group (A) country can accumulate without being rationed is: 
\[ (B_{A,1}+Z_{A,1})^\text{max} = \frac{t_{A,2}^\ast Y_{A,2} - G_{A,2}^\ast}{(1+r)} \]

Therefore, the optimal monetary policy verifies [see Appendix B and equation (5)]:
\[ \pi^* = \pi_{HB} = \bar{\pi} + \frac{\gamma c_z c + r + \lambda c}{2a^M(1-c_z)(1+z_c)^2(\gamma c_z + \lambda c) (B_{A,1} + Z_{A,1})} \]

**Proposition 1**: The less indebted type (A) countries never default. They would not default on their debt, even if the monetary authority would decide to stick to its target (\(\bar{\pi}\)) and to conduct a strict inflation targeting policy.

**Proposition 2**: If the public debt is moderate in the type (B) countries, the monetary authority can choose to accommodate its monetary policy (\(\pi > \bar{\pi}\)), which is more inflationary than expected, in order to avoid the default of these countries. The central bank pursues this
monetary policy as long as avoiding the default from some governments is worth the cost in terms of deviating from its inflation target.

**Proposition 3:** The monetary authority doesn’t accommodate to fully avoid the default of the highly indebted type (C) countries. These countries have to default on a part of their debt.

Therefore, creating a monetary union between very heterogeneous countries regarding their budgetary and fiscal situation can have harmful consequences. Indeed, in a monetary union, the fiscally strong and virtuous type (A) countries have to finance their public debt at a higher cost than if they had remained independent, because the inflation rate is then higher. On the opposite, fiscally weak type (C) countries can be forced to default at least on a part of their debt in a monetary union, without being rescued by the common central bank.

Nevertheless, the extent to which the central bank accepts to deviate from its inflation target depends on the weight given to price stability ($a^M$). Indeed, if this weight is negligible ($a^M \rightarrow 0$), the central bank tends to accommodate its monetary policy to the most fiscally weak countries ($\pi = \pi_{HC}$), and the inflation rate is then very high. On the contrary, if the weight given to price stability is very high ($a^M \rightarrow \infty$), the central bank fixes the inflation rate at the level corresponding to its target ($\pi = \pi_\pi$), even if some countries have then to default.

The optimal monetary policy also depends on the relative size of the fiscally weak type (C) countries in the monetary union ($\gamma_C$). Indeed, the more these fiscally weak countries are numerous in the monetary union, the more the central bank tends to deviate from the targeted level of inflation and to accommodate its monetary policy. On the contrary, the monetary authority is less likely to have an inflationary policy to improve the situation of a single small country in a bad fiscal position.

3. **The factors of a high inflation rate**

Beyond a weak preference of the central bank for price stability ($a^M$) or the presence of a high number of very indebted countries ($\gamma_C$), according to equation (20), there are other parameters of a high inflation rate in a monetary union.

First, the optimal inflation rate is an increasing function of the share of nominal debt ($B_{C,1}$) of the fiscally weak type (C) countries in the global level of indebtedness ($B_1+Z_1$) of the monetary union. Indeed, a high inflation rate can contribute to lighten the real cost of servicing the public debt for a country (i) which has a high share of nominal debt, whereas it is useless in case of inflation indexed debt. Therefore, the higher the share of inflation indexed debt for the fiscally weak type (C) countries, the less the common central bank has interest in deviating from its inflation target ($\pi$). In a way, as mentioned in section III.1, the share of the debt which is issued in inflation indexed bonds can be a good indicator of the fiscal soundness of a country. Indeed, only the most virtuous countries, which do not need a high inflation in order to decrease the cost of servicing a high nominal debt, have interest in issuing such inflation indexed bonds.

The optimal inflation rate is also an increasing function of the real interest rate on the risk free capital ($r$). Indeed, if this interest rate is very high, the investors ask for a high yield to service the debt. Therefore, the risks of default are sizeable, and the inflation rate must be very high in order to avoid the default of some countries. On the contrary, if this interest rate is weak, the service of the debt is less costly. Therefore, the inflation rate necessary to avoid the default of some countries can be smaller.
The optimal inflation rate also increases as the incentive effect on the issuing of public debt of a smaller cost of default \([\lambda(c-c_C)>0]\) decreases in the fiscally weak type (C) countries. In these conditions, the optimal inflation rate is an increasing function of the cost of default on the debt \((c_C)^4\) in the most indebted type (C) countries (see Figure 2). This function can even become exponential, if this cost becomes very high. Indeed, the higher this cost of default, the more harmful this default is for a given government. Therefore, the common central bank could then be tempted to accept a higher inflation rate in order to avoid the hard consequences of a default from the most indebted type (C) countries.

**Figure 2: Optimal inflation rate and cost of default**

On the contrary, the optimal inflation rate is a slightly decreasing function of the interest rate on the inflation indexed bonds in the fiscally weak type (C) countries \((z_C)^5\) (see Figure 3). Indeed, the higher this interest rate, the higher are the risks of default, because of the cost of the debt repayment for the type (C) countries. If the central bank gives a non-negligible weight to price stability, it has then interest in letting type (C) countries defaulting on a share of their debt growing with the interest rate \((z_C)\), as an accommodative monetary policy would not be worse the deviation from its inflation target. Indeed, a higher inflation rate would not help to inflate away the high indebtedness of type (C) countries.

The figures suppose the following calibration: \(B_{C,1}=(B_1+Z_1)=0.1; \gamma_C=0.05; r=0.025; a^M=2; \bar{\pi}=0.02; c=0.5; \lambda=2.\) Moreover, \(c_C=0.47\) for figure 3, \(z_C=0.06\) for figure 2.

\[\frac{d\pi}{dc_C} = \frac{\gamma_C B_{C,1}(1+z_C)(\gamma_C+\lambda c_C-\lambda c_C^2)+z_C(1+r)(1-c_C)^2}{2a^M(1-c_C)^2(1+z_C)^2(\gamma_C+\lambda c_C+\lambda c_C^2)(B_1+Z_1)} > 0.\] The figures suppose the following calibration: \(\frac{B_{C,1}}{(B_1+Z_1)}=0.1;\)

\[\frac{d\pi}{dz_C} = \frac{-\gamma_C B_{C,1}(c_C(\gamma_C-r)+(2-c_C)(1+r))}{2a^M(1-c_C)(1+z_C)^2(\gamma_C-\lambda c_C+\lambda c_C^2)(B_1+Z_1)} < 0.\]
IV. Empirical implications

To complement the theoretical part of our paper, we can have a quick look at the concrete implications of this model. Indeed, a deeper empirical study or an econometrical perspective would go beyond the framework of the current paper. However, first empirical observations can already be used to test the predictions of our theoretical model.

Figure 4 represents the empirical evolution of the average inflation rate in the EMU, on the period 2002-2011. How can this empirical evolution be explained by the theoretical conclusions of our model? What are the implications of our model, regarding the management of the public debt levels and their consequences, for the future of the European Union?

Source: Eurostat, monthly data, annual change on consumer price index.
1. The evolution of the risk free interest rate (r)

The first prediction of our theoretical model is that the optimal inflation rate is an increasing function of the real interest rate on the risk free capital (r).

In the EMU, the interest rates on the monetary market have reached a maximum of 4.3% in August 2008. This corresponds to the maximum average inflation rate in the Euro area: 3.8% in August 2008. On the contrary, the interest rates have been particularly weak since April 2009: below 1% (see Figure 5). This is in conformity with the prediction of our model that in this case, the optimal inflation rate can be smaller. Indeed, between December 2008 and November 2010, the inflation rate was well below its target of 2% in Europe, without real inflationary tensions, despite the high levels of indebtedness of some European countries. Nevertheless, in 2010 and 2011, Figure 4 shows a gradual and constant increase in the inflation rate in the EMU, whereas the increase in the interest rates on the monetary market was much more limited. We can then deduce from these empirical observations that specific inflationary factors should have appeared since 2010…

![Figure 5: Euro area interest rates on the monetary market, 2002-2011](source: Eurostat, interest rate on the monetary market, monthly data.)

2. The respective shares of nominal and inflation indexed bonds

The second prediction of our model is that the optimal inflation rate is an increasing function of the share of nominal debt of the fiscally weak countries in the global level of indebtedness in the monetary union.

First, we can mention that in the framework of our model, the common monetary authority can only conduct a credible inflation targeting policy (\(\pi = \bar{\pi}\)) if all member countries of the monetary union are weakly indebted, that is to say, if they are type (A) countries which have no risk to default. A credible inflation targeting policy implies that type (B) and (C) countries cannot exist (see Appendix C). Therefore, debt ceilings, which prevent the very indebted countries from joining the monetary union, are a necessary condition for the
credibility of the anti-inflationary policy of the common central bank. So, in order to preserve
the price stability aim of the common central bank, there should be a limit \( \text{ex ante} \) to the level
of indebtedness of the countries who want to join the monetary union. Very indebted
countries should not be allowed to become members, and a potential member country \( i \)
should have a public debt smaller than: 
\[
(B_{i1} + Z_{i1}) \leq \frac{t_{i2} Y_{i2} - G_{i2}}{(1 + z_i)}.
\]

Besides, beyond the question of the indebtedness of the member countries of a
monetary union, our model shows that the structure of this public debt, nominal or inflation
indexed, is of the highest importance. De Haan and Wolswijk (2005) mention that inflation
indexed bonds have many advantages. In particular, for a government, they can contribute to
stabilize its real public expenditures and thus to tax smoothing, especially in case of demand
shocks, as inflation and growth are then positively correlated. They are attractive for inflation
adverse investors like pension funds, and they reduce the incentives for governments to put
pressure on the central bank to accommodate its monetary policy with a higher inflation rate
intended to reduce the real value of their debt. However, they also imply a larger sensitivity of
the budget to inflation, and thus a reduced predictability of nominal interest payments, as well
as risks of segmenting the market, and perhaps also of a higher default risk premium.

Furthermore, the market for inflation indexed bonds is growing rapidly in Europe
(Garcia and Van Rixtel, 2007). To issue such bonds, the current aim is not to have a
protection against inflation, as paradoxically, this market is growing whereas inflation
expectations have become particularly weak. It is mostly a way, for the investors and in
particular for pension funds, to achieve a better portfolio diversification. The United-Kingdom
has issued inflation indexed bonds since 1981; today, they represent about a quarter of UK
government debt, around 200 billions of Euros. Sweden has also issued inflation indexed
bonds since 1994, but for much smaller amounts: about 20 billions of Euros today. Within the
Euro area, France (since 1998), Greece and Italy (since 2003) have been the first countries to
issue inflation indexed bonds. Nevertheless, even if the number of issuing countries is
growing (Germany since 2006), the share of inflation indexed bonds in total government debt
is still very small in the Euro-area countries: about 10\% for France, 3\% for Italy, 2\% for
Greece, and 0.3\% for Germany. Therefore, according to our model, the obvious dominance of
nominal debt in the Euro-area can only reinforce the inflationary tensions due to the
budgetary situation of the most indebted countries.

3. The cost of default (\( c_c \))

The third prediction of our model is that the optimal inflation rate is an increasing
function of the cost of default in the most indebted countries.

Regarding this criterion, table 1 estimates the value of the differential in the cost of
default on the debt: \( \lambda (c_{i} - c) \) for the EMU member countries, with the help of equation (5).
Then, this table shows that these differentials would be empirically very small for the
European countries. Indeed, the cost of a default on the debt would be the highest in Spain
and would the smallest in the most heavily indebted countries, whose weight in the global
debt level is higher than their weight in the global GDP: Italy and Greece. However, the
negative differential with the average cost of default would have been much reduced for Italy
between 2002 and 2010. Therefore, this higher cost of default on the debt (\( c_c \)) in Italy could
be one of the factors of a higher inflation rate in the EMU in 2008 or in 2010. Nevertheless,
we can wonder whether the default on the debt of one member country is a real option, for the Economic and Monetary Union.

**Table 1: Estimations of: $\lambda(c-c)$**

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Spain</td>
<td>0.023</td>
<td>0.051</td>
<td>0.038</td>
<td>0.033</td>
<td>Slovenia</td>
<td>0.002</td>
<td>0.003</td>
<td>0.002</td>
</tr>
<tr>
<td>Netherlands</td>
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<td>0.011</td>
<td>0.015</td>
<td>0.017</td>
<td>0.000</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>Finland</td>
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<td>0.010</td>
<td>0.009</td>
<td>0.009</td>
<td>Malta</td>
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<td>0.000</td>
<td>0.000</td>
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<td>France</td>
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<td>0.002</td>
<td>0.008</td>
<td>Ireland</td>
<td>0.010</td>
<td>0.007</td>
<td>0.003</td>
</tr>
<tr>
<td>Germany</td>
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<td>0.018</td>
<td>0.008</td>
<td>Portugal</td>
<td>0.004</td>
<td>0.000</td>
<td>-0.001</td>
</tr>
<tr>
<td>Austria</td>
<td>0.001</td>
<td>0.003</td>
<td>0.004</td>
<td>0.005</td>
<td>Belgium</td>
<td>-0.019</td>
<td>-0.010</td>
<td>-0.008</td>
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<tr>
<td>Slovakia</td>
<td>0.002</td>
<td>0.004</td>
<td>0.004</td>
<td>0.004</td>
<td>Greece</td>
<td>-0.011</td>
<td>-0.015</td>
<td>-0.016</td>
</tr>
<tr>
<td>Luxembourg</td>
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<td>0.003</td>
<td>0.003</td>
<td>0.003</td>
<td>Italy</td>
<td>-0.097</td>
<td>-0.087</td>
<td>-0.076</td>
</tr>
</tbody>
</table>

Source: own calculations; *Eurostat* data for the relative debt levels and the relative GDP.

On a strict legal point of view, the European Treaties prevent bailouts and the fact that a government buys the debt liabilities of another country in order to ease its sovereign debt burden. Indeed, article 123 on the Functioning of the European Union prohibits monetary financing:

“Overdraft facilities or any other type of credit facility with the European Central Bank or with the central banks of the Member States in favor of Union institutions, bodies, offices or agencies, central governments, regional, local or other public authorities (...) shall be prohibited, as shall the purchase directly from them by the European Central Bank or national central banks of debt instruments”.

Furthermore, article 125 expresses the no bail-out clause:

“A Member State shall not be liable for or assume the commitments of central governments, regional, local or other public authorities, other bodies governed by public law, or public undertakings of another Member State, without prejudice to mutual financial guarantees for the joint execution of a specific project”.

Nevertheless, during the spring of 2010, the European governments have agreed on a bailout package to help individual countries struggling to pay back their debts. Sovereign debts have been transferred for the first time between member States within the EU. On 2 May 2010, the governments agreed on a rescue package of bilateral loans especially designed for Greece (€80 billion from the EU countries and €30 billion from the International Monetary Fund), conditioned on the country pursuing a series of reforms and austerity measures. It has been assumed that these loans could not be assimilated to a bailout, as they are repayable with an interest rate of around 5%. Furthermore, credit lines were created for any Euro-zone country in difficulties to pay back its debt. First, the European Commission is entitled to raise a ‘European Stabilization Fund’ representing up to €60 billion, by issuing bonds at low interest rates, in order to lend afterwards this money to countries in difficulties, the EU budget serving as collateral. Moreover, a separate ‘European Financial Stability Facility’ of bilateral loans guarantees, establishing a Special Purpose Vehicle which can issue bonds worth up to €440 billion, is also created on 7 June 2010.

As these first measures prove to be insufficient to solve the Greek debt crisis, on 21 February 2012, the Finance ministers from the 17 Euro-zone countries agreed on a bailout deal and on a second rescue plan. Greece stands to receive €130 billion in loans as well as a €107 billion debt reduction, in order to avoid a massive default of the country. According to the IMF, these measures were supposed to reduce Greece's debt to annual GDP ratio to 120.5% by 2020 (against about 160% in 2012). In parallel to this agreement, Greece’s private
creditors agreed on a voluntary debt restructuring process. The private sector bond holders have to accept a higher than expected loss of 53.5% on their Greek bonds holdings (in our model, this would imply a cost of default: \( c_c = 0.46 \)). However, the bail-out loans were to be paid into a special ‘escrow’ account that would be used to pay creditors rather than to meet Greece's internal public expenditures, implying for Greece a loss of sovereignty. Moreover, in exchange for the bailout, Greece will have a near permanent task force present from the European Commission to make sure that the proper reforms are carried out.

Finally, the European Stability Mechanism (ESM) is a permanent rescue funding, liable to succeed to the previous temporary funding mechanisms in the 17 Euro-zone countries after 2013. All these measures prove that in fact, the possibility of default of one EMU member country is an extreme option, for the European governments and political decision-makers. Nobody wants to consider this option and therefore, everything is made to avoid contemplating the consequences of such a situation. Nevertheless, longer-term doubts over the ability of the Greek government to repay its outstanding debt remain, raising questions about whether even more rescue money will eventually be needed. Today, the default of one EMU member country is no longer an unbelievable option.

4. Interest rates on inflation indexed bonds (\( z_c \))

The fourth prediction of our model is that the optimal inflation rate is a slightly decreasing function of the interest rate on inflation indexed bonds in the fiscally weak countries. What are the empirical observations regarding this criterion?

Garcia and Van Rixtel (2007) show that in fact, there is a co-movement between nominal and inflation indexed bonds yields, the former being about 2% higher than the latter [see also equation (1)]. In this framework, table 2 shows that for all European countries, the government bond yields decreased from 2002 to 2005, and then increased until 2007 with the rise in interest rates on the monetary market (see Figure 5). Afterwards, the variations in government bond yields have diverged between the European countries. These yields have decreased between 2007 and 2010 for most European countries, in conformity with the decrease in interest rates on the monetary market. However, they remained constant in Spain, and they even increased in the European countries with the toughest budgetary and public debt difficulties: Greece, Ireland and Portugal.

Table 2: Government bond yields, 10 years’ maturity, annual data

<table>
<thead>
<tr>
<th></th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
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<th>2008</th>
<th>2009</th>
<th>2010</th>
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</thead>
<tbody>
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<td>Germany</td>
<td>4.8</td>
<td>4.1</td>
<td>4</td>
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<td>3.8</td>
<td>4.2</td>
<td>4</td>
<td>3.2</td>
<td>2.7</td>
</tr>
<tr>
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<td>4.1</td>
<td>4.1</td>
<td>3.4</td>
<td>3.8</td>
<td>4.3</td>
<td>4.4</td>
<td>3.9</td>
<td>3.2</td>
</tr>
<tr>
<td>Belgium</td>
<td>5</td>
<td>4.2</td>
<td>4.2</td>
<td>3.4</td>
<td>3.8</td>
<td>4.3</td>
<td>4.4</td>
<td>3.9</td>
<td>3.5</td>
</tr>
<tr>
<td>Spain</td>
<td>5</td>
<td>4.1</td>
<td>4.1</td>
<td>3.4</td>
<td>3.8</td>
<td>4.3</td>
<td>4.4</td>
<td>4</td>
<td>4.3</td>
</tr>
<tr>
<td>Italy</td>
<td>5</td>
<td>4.3</td>
<td>4.3</td>
<td>3.6</td>
<td>4.1</td>
<td>4.5</td>
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<td>4</td>
</tr>
<tr>
<td>Ireland</td>
<td>5</td>
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<td>4.1</td>
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<td>4.5</td>
<td>5.2</td>
<td>5.7</td>
</tr>
<tr>
<td>France</td>
<td>4.9</td>
<td>4.1</td>
<td>4.1</td>
<td>3.4</td>
<td>3.8</td>
<td>4.3</td>
<td>4.2</td>
<td>3.7</td>
<td>3.1</td>
</tr>
<tr>
<td>Greece</td>
<td>5.1</td>
<td>4.3</td>
<td>4.3</td>
<td>3.6</td>
<td>4.1</td>
<td>4.5</td>
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</tr>
<tr>
<td>Netherlands</td>
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<td>4.1</td>
<td>4.1</td>
<td>3.4</td>
<td>3.8</td>
<td>4.3</td>
<td>4.2</td>
<td>3.7</td>
<td>3</td>
</tr>
<tr>
<td>Portugal</td>
<td>5</td>
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<td>4.1</td>
<td>3.4</td>
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<td>4.4</td>
<td>4.5</td>
<td>4.2</td>
<td>5.4</td>
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</tbody>
</table>

Source: AMECO
More precisely, since 2008, government bond yields are much higher in Greece, in Ireland and in Portugal. Therefore, according to our model, these high interest rates on the public debt of three EMU countries with the toughest budgetary difficulties can contribute to moderate the inflationary tensions in the monetary union. However, they also increase the probability of default on a share of their debt of these fiscally weak European countries [see equation (12)]. Indeed, during the spring 2010, the fast growth in the interest rates on the Greek public debt caused its financing difficulties and the anxiety of the financial markets about the solvability of the Greek government, even if they could also have contributed to maintain a moderate inflation rate in the Economic and Monetary Union. According to our model, without very high government bond yields in Greece, the inflationary tensions in the monetary union could have been much more accentuated than those observed in Figure 4.

V. Conclusion

In our model, if some countries have very high levels of indebtedness, the monetary authority faces a trade-off between inflation and the default risk. In this case, in a closed economy, a national central bank could be tempted to ease its monetary policy in response to the budgetary difficulties of the national government. But in a monetary union with many countries only one of which has budgetary difficulties, the marginal benefit of inflation is reduced. The monetary policy of the common central bank is therefore less accommodative (high interest rates and low inflation) in case of a fiscal crisis, and the costs of servicing the debt are higher. In fact, the budgetary constraint is thus tighter for the member countries of a monetary union.

Our model shows that the common central bank of a monetary union tends to conduct a more accommodative monetary policy in order to avoid the default of the member countries whose levels of indebtedness remain moderate. On the contrary, the most heavily indebted member countries have to default at least partially on their debt, since, for the central bank, deviating from its inflation target costs more than the benefits made through avoiding the default of one country. More precisely, our model shows that in a monetary union, the optimal inflation rate increases as the weight given by the common central bank to the aim of price stability gets smaller, as the number of highly indebted countries in the union grows, and as the share of nominal debt of these countries in the global indebtedness of the monetary union gets higher. So, in the EMU for example, even if the market for inflation indexed bonds is growing today, nominal debt remains widely dominant: this reinforces the inflationary tensions due to the high level of indebtedness of some member countries. Besides, the optimal inflation rate increases as the interest rate on the risk free capital gets higher and as the interest rate on inflation indexed bonds in the fiscally weak countries is reduced. Indeed, the higher and growing interest rates on the public debt of Greece have largely increased the probability of default of this country during the spring of 2010, even if they have also contributed to maintain a moderate inflation rate in the monetary union. Finally, the optimal inflation rate increases as the incentive effect on the issuing of public debt of a smaller cost of default decreases, and then it is an increasing function of this cost of default in the fiscally weak countries.

Appendix A: Optimal debt portfolio
The definition of the group (C) countries implies:

\[
\pi \leq \pi - \left(1 + c_B - c_C\right)\frac{Z_{B,1}}{B_{C,1}}
\]

(1) and (7) imply:

\[
t_{i,2} V_{i,2} - G_{i,2} = [c_i(1 + z_i) + (1 + r)(1 - c_i)](B_{i,1} + Z_{i,1})
\]

(A1)

The group (B) implies:

\[
\pi - \left(1 + c_B - c_C\right)\frac{Z_{B,1}}{B_{B,1}} \left(1 + c_B - c_C\right)\frac{Z_{B,1}}{B_{B,1}} < \bar{\pi} < \pi - (1 + c_B)\frac{Z_{B,1}}{B_{B,1}}
\]

(A2)

The definition of the group (A) countries implies:

\[
\pi \geq \pi + (1 - c_A)(z_A - r) \left(1 + \frac{Z_{A,1}}{B_{A,1}}\right)
\]

Thus:

\[
\pi - \left(1 + c_B - c_C\right)\frac{Z_{B,1}}{B_{B,1}} \left(1 + c_B - c_C\right)\frac{Z_{B,1}}{B_{B,1}} < \bar{\pi} \leq \pi - (1 + c_B - c_C)\frac{Z_{C,1}}{B_{C,1}}
\]

(A2)

Therefore, with the expression of \(c_i\) incompatible with the definition of these countries.

\[
\pi_{LB} - \pi_{LA} = (1 + r) \left(\frac{1 - c_A}{c_A} + \frac{Z_{A,1}}{B_{A,1}} - \frac{1 - c_B}{c_B} + \frac{Z_{B,1}}{B_{B,1}}\right) > 0
\]

\[
\pi_{LC} - \pi_{LB} = (1 + r) \left(\frac{1 - c_B}{c_B} + \frac{Z_{B,1}}{B_{B,1}} - \frac{1 - c_C}{c_C} + \frac{Z_{C,1}}{B_{C,1}}\right) > 0
\]

\[
\pi_{HA} - \pi_{LC} = (1 + r)(1 - c_c) \left(1 + \frac{Z_{C,1}}{B_{C,1}}\right) > 0 \quad \pi_{HB} - \pi_{HA} = (1 - c_B)(z_B - r) \left(1 + \frac{Z_{B,1}}{B_{B,1}}\right) \geq 0
\]

\[
\pi_{HC} - \pi_{HB} = (1 - c_c)(z_c - r) \left(1 + \frac{Z_{C,1}}{B_{C,1}}\right) - (1 - c_B)(z_B - r) \left(1 + \frac{Z_{B,1}}{B_{B,1}}\right) > 0 \quad \text{as } z_B = r \text{ if } \pi \geq \pi_{HB}.
\]

(A3)

Therefore, with the expression of \(c_i\) mentioned in (5), (A3) implies:

\[
\frac{Z_{A,1}}{B_{A,1}} > \frac{(c_A - c_B)}{c_B(1 - c_A)} + \frac{c_A(1 - c_B)}{B_{B,1}} Z_{B,1} \quad \frac{Z_{B,1}}{B_{B,1}} > \frac{(c_B - c_C)}{c_C(1 - c_B)} + \frac{c_B(1 - c_C)}{B_{C,1}} Z_{C,1}
\]

Appendix B: Optimal monetary policy

- If \(\pi < \pi_{HA}\): type (A) countries would default on a part of their debt, which is incompatible with the definition of these countries.
- If \(\pi \in [\pi_{HA}, \pi_{HB}]\): type (A) countries never default; types (B) and (C) countries default on a part of their debt.

Using the definition of the type (A) countries, we should get:

\[
\pi_{HA} = \pi = \pi < \pi_{HB} = \pi + (1 - c_B)(z_B - r) \left(1 + \frac{Z_{B,1}}{B_{B,1}}\right)
\]

In the same way, if: \((y_B - y_C)(B_1 + Z_1) + (B_{C,1} + Z_{C,1}) - (B_{B,1} + Z_{B,1}) > 0\), we have:

\[
\frac{Z_{A,1}}{B_{A,1}} > \frac{Z_{B,1}}{B_{B,1}}
\]

In the same way, if: \((y_B - y_C)(B_1 + Z_1) + (B_{C,1} + Z_{C,1}) - (B_{B,1} + Z_{B,1}) > 0\), we have:

\[
\frac{Z_{B,1}}{B_{B,1}} > \frac{Z_{C,1}}{B_{C,1}}
\]
However, this policy is then optimal only if $\pi=\bar{\pi}$, that is to say if: $\frac{\partial d_i^*}{\partial \pi}=0$ ($\forall i$), which is not the case in this situation, as at least for the type (C) countries, the probability of default on their debt depends on the inflation rate.

- If $\pi\in[\pi_{HB};\pi_{HC}]$: types (A) and (B) countries never default; type (C) countries default on a part of their debt. Thus, we have $z_A=z_B=r$.

Thus: $\bar{\pi} \leq \pi - \frac{1+r(1-c_C)}{c_C} \left( 1 + \frac{z_{C,1}}{B_{C,1}} \right) \leq \pi_{HB} = \pi \leq \pi_{HC} = \pi + (1-c_C)(z_C - r) \left( 1 + \frac{z_{C,1}}{B_{C,1}} \right)$

$\pi^* = \bar{\pi} - \frac{1}{2a_M} \frac{\partial d_i^*}{\partial \pi} = \bar{\pi} + \frac{y_C (t_C - G_{C,2}) B_{C,1}}{2a_M (1-c_C)((1+b_C) - \pi)B_{C,1} + (1+z_C)Z_{C,1}^2}$

- If $\pi>\pi_{HC}$: Types (A), (B) and (C) countries never default on their debt. Therefore, we have $\frac{\partial d_i^*}{\partial \pi}=0$ ($\forall i$), and $\pi=\bar{\pi}$.

This implies: $\bar{\pi} = \pi > \pi_{HC} = \pi + (1-c_C)(z_C - r) \left( 1 + \frac{z_{C,1}}{B_{C,1}} \right)$.

However, this is incompatible with the definition of the countries from the group (C).

The optimal monetary policy is therefore: $\pi^* = \pi_{HB} = \bar{\pi}$, where the type (C) countries are the only one to default on their debt. With (A1) we have:

$\pi^* = \pi_{HB} = \bar{\pi} + \frac{y_C (1+r + c_C z_C - r c_C) B_{C,1}}{2a_M (1-c_C)((1+z_C)^2(B_{C,1} + Z_{C,1}^2)}$

**Appendix C: Credible inflation targeting policy**

A credible inflation targeting policy implies: $\pi=\bar{\pi}$.

Type (C) countries could not exist, as they would be sure to default ($d_C=1$), and thus, as nobody would want to buy their debt.

$\pi_{LB} - \pi_{LA} = (1+r) \left[ \frac{1-c_A}{c_A} \frac{Z_{A,1}}{B_{A,1}} - \frac{1-c_B}{c_B} \frac{Z_{B,1}}{B_{B,1}} \right] > 0$ see Appendix A

$\pi_{HA} - \pi_{LB} = \frac{1+r(1-c_B)}{c_B} \left( 1 + \frac{Z_{B,1}}{B_{B,1}} \right) > 0$

$\pi_{HB} - \pi_{HA} = (1-c_B)(z_B - r) \left( 1 + \frac{Z_{B,1}}{B_{B,1}} \right) \geq 0$

- If $\pi<\pi_{HA}$: type (A) countries would default on a part of their debt, which is incompatible with the definition of these countries.

- If $\pi\in[\pi_{HA};\pi_{HB}]$: type (A) countries never default; type (B) countries default on a part of their debt. $\pi_{HA} = \pi = \bar{\pi} < \pi_{HB} = \pi + (1-c_B)(z_B - r) \left( 1 + \frac{Z_{B,1}}{B_{B,1}} \right)$

- If $\pi>\pi_{HB}$: Types (A) and (B) countries never default on their debt.

This implies: $\pi = \bar{\pi} > \pi_{HB} = \pi + (1-c_B)(z_B - r) \left( 1 + \frac{Z_{B,1}}{B_{B,1}} \right)$

However, this is incompatible with the definition of the countries from the group (B).

The optimal monetary policy is therefore: $\pi^* = \pi = \pi_{HA}$, where type (B) countries default on a part of their debt.
\[
\pi^* = \pi_{HA} = \bar{\bar{\pi}} = \bar{\bar{\pi}} - \frac{1}{2aM} \frac{\partial d_B^*}{\partial \pi} = \bar{\bar{\pi}} + \frac{1}{2aM} \frac{\partial d_B^*}{\partial \pi} = \bar{\bar{\pi}} + \frac{y_B(1 + r + c_B z_B - r c_B) B_{B,1}}{2aM \left(1 - c_B \right) \left(1 + z_B \right)^2 \left(B_{B,1} + Z_{B,1} \right)}
\]

Thus, \( \gamma_B = 0 \), and the type (B) countries can’t exist.

References