



Three Essays on Risk Modelling and Empirical Asset Pricing¹

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This dissertation consists of three papers that focus on risk modelling and empirical asset pricing. Specifically, the first paper contributes to the literature by providing a method of obtaining more efficient estimates and forecasts of covariance matrices. The second paper identifies common risk factors in panels of volatilities that drives the distribution of asset returns, and the third paper introduces basic quantile asset pricing equation with an application to factor pricing. All papers result from the natural collaboration with my supervisor Jozef Barunik who is also co-author of the papers. Therefore, in the rest of the text I stick to "we" when referring to the author. A short summary of the papers follows.

In **Chapter 1 On the modelling and forecasting multivariate realized volatility: Generalized Heterogeneous Autoregressive (GEAR) model**, we introduce a multivariate extension of the popular Heterogeneous Autoregressive model. This paper is published in *Journal of Forecasting* (Cech and Barunik, 2017).

Volatility modeling and forecasting are key issues in the area of financial econometrics. In empirical work, researchers and practitioners often study stock market data and find dependencies in the second moment of these data. As shown by Engle (1982), Bollerslev (1986), Nelson (1991) and many others, volatility of financial time series is anything but constant. To deal with this problem, a new family of parametric univariate conditionally heteroscedastic models represented by the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) was developed in the eighties and nineties.

While the search for more accurate volatility models has been the focal point of many researchers, interdependencies among assets and subsequent comovements are of great importance in practice (e.g., asset allocation, portfolio management, risk management, etc.). The natural extension of the family of volatility models is to model the whole covariance structure of the given assets. This gives rise to the development of the multivariate GARCH models. Although the transition from univariate to multivariate GARCH models might seem to be straightforward, it possesses several challenges. Multivariate volatility modeling nowadays offers numerous research opportunities in the form of extension to, or innovation of, current methodologies; as well as developing techniques for solving drawbacks of current approaches (e.g., reduction of dimensionality). Our research contributes to these efforts by introducing a generalization of Heterogeneous Autoregressive (HAR) Model of Corsi (2009).

Increased availability of high-frequency data in the last decade resulted in the development of the new non-parametric approach of treating

volatility. In particular, model-free estimator of Realized Volatility in Andersen et al. (2001) makes volatility observable. Theoretical properties of this estimator have been further studied in Andersen et al. (2003) and Barndorff-Nielsen and Shephard (2004). Barndorff-Nielsen and Shephard (2004) moreover introduce the concept of Realized Covariation, which is a multivariate extension of Realized Volatility. Market microstructure noise can significantly affect Realized Covariance estimates resulting in not positive semi-definite matrices. A solution to this problem is offered by Barndorff-Nielsen et al. (2011) and their Multivariate Realized Kernels estimator that guarantees the positive semi-definiteness of the covariance matrix.

All the realized measures, univariate or multivariate, are ex-post measures of return (co)variation. These measures need to be further modeled, so they are of some practical use. The research devoted to entire covariance structure modelling is ongoing and growing. Part of the researchers makes use of variants of the Wishart distribution to model the structure of the realized covariances (Gouriieroux et al., 2009; Bonato, 2009; Bonato et al., 2013; Jin and Maheu, 2013). Another stream of researchers decompose realized covariance matrices by matrix exponential-logarithm transformation or Cholesky decomposition and use standard time-series techniques afterwards (Bauer and Vorkink, 2011; Chiriac and Voev, 2011). The advantage of the decomposition approach is a guarantee of the positive-semidefiniteness of the covariance matrix forecasts. Our work contributes to the literature by introducing Generalized Heterogeneous Autoregressive Model (GHAR), a multivariate extension of the popular HAR Model intended for covariance matrix modelling and forecasting.

In our work, we stick to the covariance decomposition stream of literature. Specifically, we model Cholesky factors of the realized covariance matrices as a system of the seemingly unrelated heterogeneous autoregressions. Motivation to

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build a system of the seemingly unrelated regression (Zellner, 1962) over HAR is the contemporaneous correlation in the residuals of the simple HAR model. The advantage of this approach is that we estimate a multivariate HAR model, which will capture the separate dynamics of the variances and covariances, but also possible common structure. Moreover, it will also yield more efficient estimates – the error terms from simple HAR are heteroscedastic (Corsi et al., 2008), which makes the coefficient estimates less efficient. Furthermore, when there is no information about dependence between equations left in the residuals estimator will converge to a simple Ordinary Least Squares (OLS) estimates, as the diagonal weighting matrix in generalized regression will reduce the estimates to OLS. Therefore, using generalized least squares, we capture dependencies hidden in the residuals delivering more efficient estimates.

In the empirical application, we study portfolios consisting of five, ten and fifteen highly liquid stocks from New York Stock Exchange, e.g. Apple Inc. (AAPL), Exxon Mobile Corp. (XOM), Google Inc. (GOOG) etc.. We begin our analysis with the one-step-ahead forecasts for the portfolio consisting of five stocks, whereas we leave portfolios of ten and fifteen stocks and also five- and ten-step ahead forecasts as a robustness check showing that the proposed methodology also works well at larger dimensions and different forecasting horizons. Our dataset consists of tick data and covers the period of Global Financial Crisis, i.e. July, 1 2005 to January, 3 2012 with 1623 trading days. As it is standard in the literature we explicitly exclude weekends and bank holidays (New Year's Day, Independence Day, Thanksgiving Day, Christmas) to ensure sufficient liquidity. From the tick data, we calculate realized covariance matrices using Multivariate Realized Kernels estimator at one-minute frequency, Realized Covariance at one and five-minute frequencies and Sub-Sampled Realized Covariance at five, ten, fifteen and twenty minute frequency. Moreover, we calculate also open-close daily returns. The model estimation and forecasting exercise are carried out using rolling window estimation with a fixed length of 750 days, i.e. three years.

We compare the performance of the GHAR against two covariance based benchmark models (HAR, Vector ARFIMA (Chiriac and Voev, 2011)), and two return based benchmarks (Dynamic Conditional Correlation GARCH (Engle, 2002), RiskMetrics (Longerstaeay and Spencer, 1996)) primarily according to economic criteria, i.e. Mean-Variance efficient portfolio of Markowitz (1952) and Global Minimum Variance Portfolio (GMVP) using cumulative and annualized risk. The rationale behind is the importance of well-conditioned and invertible forecasts rather than focusing on unbiasedness, as an unbiased forecast does not necessarily translate into an unbiased inverse (Bauwens et al., 2012). As a robustness check we also provide a ranking of the models based on the Root Mean Squared Error (RMSE) loss functions

based on the Frobenius norm and to test the significant differences of competing models, we use the Model Confidence Set (MCS) methodology of Hansen et al. (2011). The MCS procedure sequentially eliminates the worst-performing model from the full set of competing models when the null about the same forecasting performance is rejected.

Overall, the results of our analysis suggest that GHAR provides more precise and more efficient covariance matrix forecasts and they translates to economic gains directly. Specifically, in the one-step-ahead forecasting exercise using portfolio of five stocks, the GHAR shows the best performance according to all economic criteria, i.e. GHAR achieves the best risk-return trade-off in Markowitz optimization, and has the lowest risk according to both cumulative and annualized versions of GMVP. The robustness check, the portfolio of ten/fifteen stocks and five/ten-step-ahead forecasts qualitatively match our previous findings. Moreover, we document the economic benefit of estimating the realized covariance with more efficient multivariate realized kernel and sub-sampled realized covariance estimators using ten to twenty minutes sub-sampling. In the statistical comparison, we obtain a bit mixed results. While in the one-step-ahead forecasts GHAR always belongs to MCS in case of the portfolio of five and ten stocks, it is in MSC only when 5-minutes RCOV is used in case of fifteen stocks portfolio. For the forecasting horizon of five/ten days results do not change substantially. The only notable difference is absence of GHAR in MCS in the case of ten-step ahead forecasts of portfolio consisting of fifteen stocks. We address unambiguous results of the statistical evaluation to a problem of selecting the "correct" proxy for unobservable "true" covariance matrix.

In **Chapter 2 Measurement of Common Risk Factors in Tails: A Panel Quantile Regression Model for Returns**, we introduce an innovative approach of modelling commonalities in the quantiles of future returns using information from panels of realized measures. The earlier version of the paper was published in Institute of Economic Studies Working Paper series as *IES Working Paper 20/2017* and current status of the paper is *revise & resubmit* in the *Journal of Financial Markets*.

During the last two decades, global financial markets were hit by several crises. The most well-known and important ones are Dot-com Bubble and the Global Financial Crisis of 2007-2012 that includes Icelandic financial crisis and European sovereign debt crisis. The aftermath of these events highlights the necessity of proper risk identification and mitigation. The need for accurate risk measures is important not only from the regulatory point of view to prevent a future crisis but is also crucial for many applications within portfolio and risk management. Recently, the increased availability of high-frequency data resulted in the development of the more accu-



rate volatility estimators commonly referred as Realized Measures. Whether is original Realized Volatility (Andersen et al., 2001), Realized Semi-variance (Barndorff-Nielsen et al., 2010), for which the sign of the price change matters, or the adjusted Bi-Power Variation (Andersen et al., 2011) that is robust to jumps in the prices and the certain types of microstructure noise, all these realized measures help us to understand the nature of the data, identify sources and potentially predict the risk.

Although volatility forecasting is essential for many financial applications, it does not help us to specify the conditional distribution of future returns. The classical portfolio theory rather concentrates on the risk-return relationship that has a long history and is well documented. For example in the Capital Asset Pricing Model, the risk of the asset is measured by the covariance between asset return and market return. Market return is just one of the many possible factors affecting an asset's risk. Among other factors, the volatility of the asset plays an essential role in explaining expected returns.

The classical asset pricing moreover assumes an economic agent maximizing expected utility. However, the expected utility framework might be too restrictive to describe the real/actual behavior of the economic agents. Recent studies thus assume agents to maximize their quantile utilities, e.g. de Castro and Galvao (2018).

In finance, the Conditional Autoregressive Value-at-Risk (CAViaR) model of Engle and Manganelli (2004) is one of the first examples that focus on the estimation of quantiles of various asset returns, Baur et al. (2012) use quantile autoregressions to study conditional return distributions and Cappiello et al. (2014) detects comovement between random variables with time-varying quantile regression. The work of Zikes and Barunik (2016), who combine the quantile regression framework (Koenker and Bassett Jr, 1978) with realized volatility, is another important example in this field. In their work, it has been shown that various realized measures are useful in forecasting quantiles of future returns without making assumptions about underlying conditional distributions.

While Zikes and Barunik (2016) provided an important link between future quantiles of return distribution and its past/ex-ante variation, they concentrate on the univariate time series. Effective risk diversification techniques work not only with the single conditional asset return distribution, but require a deeper knowledge of the dependencies in the joint distributions. In the standard mean-regression framework, Bollerslev et al. (2016) show that realized volatility of the financial time series share many commonalities. In the quantile regression set-up, however, there is no similar study that will try to uncover information captured in the panels of volatility series. To the best of our knowledge, there is no study dealing with estimates of conditional distribution of

return series in a multivariate setting that explores ex-post information in volatility.

In this paper, we contribute to the literature by introducing a Panel Quantile Regression Model for Returns – we propose to model the panel of assets returns via its past and/or ex-ante volatility using panel quantile regression techniques. This approach allows us to exploit common factors in volatility series that directly affect quantiles of return series. Moreover, we can control for otherwise unobserved heterogeneity among financial assets. Furthermore, using the fixed effects estimator, we can disentangle overall market risk into the systematic part and idiosyncratic risks. In a sense, we revisit a large literature connecting volatility with the cross-section of returns we model tail events of the conditional distributions via volatility.

In the empirical application, we show that the newly proposed model delivers more accurate estimates than benchmark methods using various data-sets. The gain in accuracy translates into better forecasting performance of Panel Quantile Regression Model for Returns. We test the performance of our model in a portfolio Value-at-Risk forecasting exercise where we concentrate on the statistical and economic evaluation. In the statistical comparison, we distinguish between the absolute and relative performance of the given model. The absolute performance in our work is assessed by the so-called CAViaR test of Berkowitz et al. (2011) and tests whether the model is dynamically correctly specified. For the relative performance, we employ a standard Diebold-Mariano test and we pair-wise compare all the competing models. In the economic comparison, we study Global Minimum Value-at-Risk Portfolio (GMVaRP) and the Markowitz like efficient frontiers of the Value-at-Risk – Return trade-off. The economic and relative statistical performance is tested against three benchmark models – Risk-Metrics (Longestae and Spencer, 1996) and two versions of Univariate Quantile Regression Model for Returns (Zikes and Barunik, 2016).

Our analysis starts with the well-behaved simulated data from Monte-Carlo experiments. Specifically, we simulate 29 continuous price process series using four error distributions – Multivariate normal/fat-tailed Student-t distributions both with the given correlation structure obtained from the stock market data and Univariate normal/fat-tailed Student-t distributions. In total, we run 500 simulations for each error distribution and in each simulation step we use rolling window estimation with a length of 1000 observations. The results of the Monte-Carlo simulation study shows that our model is dynamically well specified and outperforms all the benchmarks in direct statistical comparison when we use more heterogeneous data generated from the univariate error distributions.

Next, we analyze 29 highly liquid stocks such as Apple Inc. (AAPL), Amazon.com, Inc. (AMZN), Bank of America Corp (BAC), Comcast Corpora-

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tion (CMCSA), Cisco Systems, Inc. (CSCO) etc. from the New York Stock Exchange during the period July 1, 2005, to December 31, 2015. In the empirical analysis, we hypothesise that quantile of open-close returns depends on the various ex-post risk measures calculated from the tick data, i.e. Realized Variance, Realized Semivariances and Realized Bi-Power Variation. For the portfolio Value-at-Risk construction we also proxy the covariance structure by the Realized Covariance estimates. Similar to simulation study we use rolling window estimation procedure with the same window length (1000 days) for the estimation and forecasting purposes. In the in-sample analysis, we document unobserved heterogeneity in far quantiles that needs to be controlled. Moreover, all the risk measures show the asymmetric impact on the quantiles of returns, e.g. impact is higher in the bellow median quantiles. In the out-of-sample forecasting exercise, we have found that all the panel quantile regression models are dynamically correctly specified. Importantly, the panel quantile regressions consistently outperform the benchmarks in various quantiles and they are not outperformed by any of the benchmarks. From the economic point of view, newly proposed modeling strategy performs best in all but median quantiles according to GMVaRP criteria and provide us with the best Value-at-Risk – Return trade-off.

Our next step was the analysis of the common exogenous risk factor in tails of the returns distribution. We have selected ex-ante measure of the market uncertainty, widely used VIX Index which measures the expectations about the 30-day market volatility, as the exogenous factor. Results of the analysis confirm that VIX carries an important part of the information about risk that is not fully captured by any of the realized measures. Moreover, by controlling for the unobserved heterogeneity and idiosyncratic volatility, VIX proves to be a strong common factor driving the tails of the return distributions. Our findings also hold in the economic comparison where panel quantile regression model with VIX achieves the best performance using both evaluation criteria.

In the last section of the paper, we test the robustness of our previous findings using high dimensional portfolio (496 assets) consisting of the constituents of the S&P 500 index. We found that the VIX Index plays an important role in the high-dimensional application and that anticipation of the future market volatility translates directly to the conditional distribution of future returns.

Overall, the results of our analysis suggest that the Panel Quantile Regression Model for Returns is dynamically correctly specified. Moreover, it dominates benchmark models in the economically important quantiles (5%, 10% or 95%) and we find that none of the benchmark models is able to outperform our model consistently. Furthermore, the Panel Quantile Regression Model for Returns provides us with direct economic gains according to both economic evaluation criteria.

In **Chapter 3 Dynamic Quantile Model for Bond Pricing**, we concentrate on the quantile pricing of bond future contracts.

In this work, we study the bond pricing in the tails of the returns distributions. As opposed to classical asset pricing (Sharpe, 1964; Lintner, 1965; Merton, 1973; Ross, 1976), we make a step forward and move from the expected utility set-up to quantile preferences. This transition allows us to study asset pricing given the economic agents differing in their level of risk aversion. In particular, we build on work of de Castro and Galvao (2018) who derive quantile Euler equation using properties of quantile preferences as defined in Manski (1988) and Rostek (2010). We also utilize the advantages of quantile preferences such as robustness to fat tails and the ability to capture heterogeneity through the quantiles. We further extend the results of de Castro and Galvao (2018) into a stochastic discount factor representation of the quantile asset pricing equation and present a link to the factor models.

In the empirical application, we focus on quantile pricing of the two, five, ten and thirty years US and German government bond futures contracts from the Chicago Board of Trade and the EUREX exchanges. The US Treasuries dataset consists of the individual assets tick prices from the period July 1, 2003, to November 30, 2017 during regular trading hours – Sunday to Friday, 5:00 p.m. – 4:00 p.m. Chicago Time. We further consider selected maturities of US forward rates estimates to play an important role in the bond pricing as it is common in the literature, e.g. Cochrane and Piazzesi (2005). These data are obtained from the dataset of Gurkaynak et al. (2007) where the detailed estimation procedure of data creation is described. In the case of German treasury futures, we are working with tick prices from the period October 1, 2005, to November 30, 2017 during standard trading hours – Monday to Friday, 8:00 a.m. – 10:00 p.m. Central European Time. To ensure sufficient liquidity, we explicitly exclude public holidays and days with less than 5 hours of trading. From the raw tick data, we extract 5 minutes prices, and we calculate open-close returns, Realized Volatility (Andersen et al., 2003) and Realized Semi-variance (Barndorff-Nielsen et al., 2010).

For estimation purposes, we adopt recently developed smoothed (Generalized) Method of Moments quantile estimator of de Castro et al. (2018) and the quantile regression of Koenker and Bassett Jr (1978). First, we study single-factor-model with Realized Volatility being the risk factor. In this set-up, we illustrate the proximity of the GMM quantile estimator and the standard quantile regression. Second, relying on the similarity of both methods we study multi-factor-model where we rely solely on the quantile regression approach since the implementation of multiple moment conditions in quantiles is not trivial and is subject to further research. We consider two multi-factor specifications in our work. In the first specification positive and negative Realized Semivariance



serve as a risk factor. The second specification is motivated by the Cochrane and Piazzesi (2005) and we consider two to five years forward rates.

Results of our analysis demonstrate a significant influence of the Realized Volatility on the quantiles of the treasury returns. In both US and German treasuries, we obtain qualitatively similar results using both GMM and quantile regression estimators. Quantitatively, however, results differ a bit and we attribute these differences to lower liquidity of the German treasuries. When we concentrate on the comparison of GMM and quantile regression we again obtain qualitatively and also quantitatively similar results. Specifically, the majority of the GMM coefficients estimates lies almost always in the 95% confidence intervals of the quantile regression and vice versa. Interestingly, while in German treasuries quantile regression in almost all quantiles underestimates the influence of the Realized Volatility compared to GMM estimates, in the US Treasuries, GMM and quantile regression estimates intersect frequently, and there is no clear under/overestimation pattern.

In the multi-factor model, our results depend heavily on the factors used for analysis. In case Realized Semivariances are considered being risk factors results of our analysis share many commonalities with single-factor models, e.g. coefficient estimates for quantiles below/above the median have negative/positive signs, the majority of coefficients are statistically significant. Besides

similarities, we also document a unique influence of semi-variances. In the US Treasuries case, negative semivariance influence lower quantiles relatively more than the upper quantiles while the opposite is true for positive semivariance. Moreover, in the upper quantiles, positive semivariance dominates negative semivariance whereas in the lower quantiles the results are mixed. In contrast, the influence of Realized Semivariances on the quantiles of German government bonds returns is more symmetric and German treasuries look more homogeneous as the coefficient estimates closer to each other.

In the last part of the paper, we study the multi-factor model when forward rates serve as the risk factors. In this part, we concentrate on the US Treasuries only since the data for the German market are not available at the desired (daily) frequency. Our analysis shows that forward rates carry very limited information about bond returns distributions. Specifically, for all the treasuries and all the forward rates, the vast majority of the estimates is statistically insignificant. Hence, the risk-averse investor optimizing quantiles below median finds forward rates of limited use since their coefficients are not statistically different from zero. The only exception where a risk-loving investor might consider forward rates to be valid risk factors is the shortest maturity treasury where selected forward rates show partial explanatory power in the upper quantiles of the bond returns distribution.

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