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COSTS AND BENEFITS OF SLOVAKIA ENTERING THE EURO AREA

A QUANTITATIVE EVALUATION

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Costs and benefits of Slovakia entering the euro area. A quantitative evaluation.

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Abstract

Entering monetary union brings both benefits and costs. The loss of an independent monetary policy, including the loss of exchange rate policy, constrains the ability to stabilize the domestic economy in the event of asymmetric shocks. This leads to more volatile business cycles and hence lower utility of risk-averse agents in the economy. On the other hand, the common currency reduces transaction costs, thus increasing trade and growth. The objective of this article is to quantitatively evaluate these costs and benefits, using an estimated two-country DSGE model for Slovakia and the euro area.

JELclassification: E 42, E 52, F 41, F42
Key words: Monetary union, costs and benefits, two-country DSGE

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1. INTRODUCTION

The recent financial crises exposed the euro area to crucial test of long term sustainability. The original intention of creating currency union, of achieving nominal and consequently real convergence of its member states' economies, has not been fulfilled. On the contrary, economies have drifted further apart, especially due to widening gap in their competitiveness.

The objective of this article is much more subtle than trying to address the above problem. The article casts some light on some costs and benefits of Slovakia's membership in the currency block. As a tool to carry out a quantitative evaluation, we use MUSE - an estimated two country DSGE model, developed at the Research department of the National bank of Slovakia (Senaj, Vyskrabka and Zeman, 2010).

There is extensive literature on evaluating costs and benefits of joining a monetary union through the use of DSGE models. These models are a suitable tool, as one of their main component – utility function - provides a quantitative gauge that can be used to measure welfare under various scenarios, various parameter values and various shocks.

The pioneering article - Benigno (2004) - investigates optimal monetary policy in a two-region economy characterized by asymmetric shocks across regions.

In another article, Clerc et al. (2011) investigate whether and under what conditions a cost of loosing stabilizing effect of independent monetary policy can be outperformed by benefits of entering monetary union and thus gaining higher credibility. Both these articles are rather theoretical and the models used are not based on specific economies.

Söderström (2008), on the other hand, uses a Bayesian estimated DSGE model to assess whether it would be beneficial for Sweden to join the euro area. He compares the cyclicality of Swedish economy with the euro area and analyses the structure of shocks that affect both economies and their impact on them. In our paper we extend the analysis used in Söderström by including welfare effects of individual shocks.

Lama and Rabanal (2011) use their two-country DSGE model, estimated with UK, euro area data, to assess the welfare trade-off between loss of independent monetary policy and positive effect of integration in monetary union on trade and financial flows. We perform similar exercise with our model.

The paper has the following structure. In chapter 2 we summarize the structure of the estimated two-country DSGE model of Slovakia within the euro area, developed by Senaj, Vyskrabka and Zeman (2010). In chapter 3 we use welfare analysis to quantify gains that result from lower transaction costs in the monetary union versus costs associated with lower stabilization power of common monetary policy. In chapter 4 we concentrate on assessing and quantifying the loss of independent monetary policy. We compare variability of economic fundamentals in both regimes, calculate welfare losses and identify relative effects of domestic and foreign shocks, respectively. Then we run counterfactual experiment showing what would have happened if Slovakia had joined monetary union from the beginning of its existence at 1999. Chapter 5 summarizes all costs and benefits, discussed in the paper and provides some tentative conclusions.

2. THE MODEL

The economy consists of two countries: a home small economy (Slovakia) and a foreign large economy (rest of the euro area)[2]. Both have symmetric structure; in each economy there are households, intermediate firms, final goods firms and governments.

Each household maximises its discounted lifetime utility by choosing a level of real consumption and real investment that increases existing capital stock, the rate of capital utilisation, domestic bond holdings and hours worked. Because each household provides differentiated labour, it has a market power by setting its wage in a monopolistically competitive market.

There are domestic intermediate goods firms each producing differentiated goods in a monopolistically competitive market. Firms rent capital from households and hire differentiated labour services to produce differentiated goods. Because the firms produce differentiated goods in a monopolistically competitive market, they have a market power by

[2] Although shocks originated in Slovak economy do affect foreign economy in the model, their impact is negligible due to a low weight of the Slovak variables in comparison with the foreign ones.

setting their prices. Intermediate goods are tradable and we assume that there is no price discrimination between these goods being sold domestically and abroad. Hence the law of one price (LOOP) holds for each good.

There are three different types of final goods firms that combine bundles of domestically produced intermediate goods with imported intermediate goods; the firms are differentiated according to the types of non-tradable final goods they produce: private consumption goods, investment goods, and government goods. They operate in a perfectly competitive market.

The government runs a balanced budget in each period. It collects lump-sum taxes and uses the tax revenue to purchase government goods and to finance social lump-sum transfers to households. In order to stabilise inflation and output of the domestic economy, the government adjusts the short-term nominal interest rate in each period according to Taylor rule in each country.

However, this model can be easily switched to a regime with a common currency, i.e. to a currency union. In the currency union there is one monetary authority that sets interest rates for both countries.

The model tracks some 25 variables in each country, 13 of them in total are observable. It also contains 14 structural shocks: consumption preference, investment, production, technology, monetary policy and government spending shocks in each country. All data series come from the Eurostat database, are seasonally adjusted, and are expressed in per capita terms. The length of the period in the model is one quarter. The sample starts in the second quarter of 1997 and ends in the fourth quarter of 2008.

The model is parameterised in two steps. First, deep structural parameters are calibrated on the basis of some empirical steady state ratios. Second, the remaining structural parameters and standard deviations of the shocks are estimated by a Bayesian method.

The list of model equations with their short description is presented in the Appendix. For full details on model specification and calibration see Senaj, Vyskrabka and Zeman (2010)[3].

[3] MUSE: Monetary Union and Slovak Economy model

3. TRADE COSTS

Trade costs are large even between economically integrated countries. By trade costs we mean all transport costs, border-related costs and local distribution costs. According to Anderson and van Wincoop (2004), the trade costs are as large as 170% ad-valorem tax equivalent for a representative rich country. This means that trade costs add up to 170% to the original production costs.

This large number can be broken down into its components: 1.21 represents transportation costs (of which 1.11 is freight and 1.09 time cost), 1.55 is local distribution costs (retail and wholesale distribution) and 1.44 border-related costs[4] (1.08 policy barrier (tariffs), 1.07 language barrier, 1.14 currency barrier, 1.06 information costs barrier, 1.03 security barrier).

Among all costs listed above, only currency barrier costs that consist mostly of hedging costs against exchange rate risk are affected when a country joins monetary union.

Let P_F be a price of goods that includes production cost and all trade costs except currency barrier costs. Then the price importers face - $\overline{P}_F$, that incorporates also currency costs, is given by

$$\overline{P}_F = \frac{P_F}{1-\tau} \quad \text{, with } \tau = 0.12, \quad \left(\frac{1}{1-\tau} = 1.14\right).$$

Because roughly 40% of all Slovak imports and exports are bought from/sold to euro area, the trade cost τ is reduced by 40% after joining the monetary union, hence its new value becomes $\tau = 0.07$.

We can now use the model to evaluate welfare analysis of the reduction of trade cost.

Table 1 summarises the results.

4 (1+1.7)=2.7=1.21x1.55x1.44

Table 1: Welfare effects of trade cost reduction

welfare	autonomous	union CTC	union LTC	welfare gain	welfare gain
	τ=0.12	*τ=0.12*	*τ=0.07*	*(utility)*	*(percentage of consumption)*
steady state	-540.32	-540.32	-536.90	3.42	0.7
volatility	-15.4	-15.8	-15.8	-0.40	-0.08
total	-555.72	-556.12	-552.70	3.02	0.62

CTC – constant trade costs

LTC – lower trade costs

Source: *Own calculations.*

The welfare effects are divided into a steady-state effect and a volatility effect.

In steady-state, lower trade cost (LTC) reduces the price of imports, which leads to higher consumption. 3.42 utility gain corresponds to 0.7% gain of lifetime steady-state consumption[5].

On business cycle frequencies cost reduction has no effect on utility. However, inside the monetary union stabilizing effect of common monetary policy is weaker for domestic variables, which are more volatile. This results in 0.08 loss of lifetime steady state consumption. The overall effect of entering monetary union, due to trade cost, leads to 0.62% gain of steady state consumption.

4. COST OF INDEPENDENT MONETARY POLICY

Households are risk averse, they prefer stable economy. Higher volatility of economic fundamentals leads to a welfare loss. Slovak economy is a very small part of the euro area and when hit by a country specific shock, the union monetary policy is not altered much. Thus the monetary policy that can react independently to stabilise the economy after a country specific shock should be more effective, especially in small countries. The cost associated with the monetary policy independence loss depends on the importance of the

[5] Because the level of welfare has little economic meaning, utility changes are converted to consumption changes by the use of an equivalent variation method.

country specific shocks on the domestic economy. The more the economy is exposed to these shocks, the higher the resulting costs. How important are the shocks originated in the Slovak economy relative to foreign shocks? We can use the estimated model to answer this question.

4.1 VARIANCE DECOMPOSITION

Relative importance of domestic and foreign shocks can be analyzed by forecast error variance decomposition[6] of main economic fundamentals, namely the output growth rate, domestic and CPI inflation and nominal interest rate. The results are reported in table 2.

The first part of the table reports increments of the forecast error variance (in absolute terms) of each variable at different horizons. Variance of each variable changes mostly within the first 4 periods and almost stabilizes after 20 periods.

Because we want to compare relative importance of domestic and foreign shocks, we pack them in the following bundles: domestic investment, technology and production shocks into domestic supply shocks bundle, domestic consumption preference, monetary and government shocks into domestic demand bundle, all foreign shocks, namely investment, technology, consumption preference, monetary and fiscal into foreign shocks bundle and finally the risk premium and inflation target shocks into other shocks bundle.

[6] It is calculated from impulse-response functions to each individual shock

Table 2: Variance decomposition

	GDP growth rate	CPI inflation	Domestic inflation	Interest rate
1) Variance				
1	0.01217	0.00037	0.00043	0.00005
4	0.01529	0.00010	0.00021	0.00007
20	0.00219	0.00000	0.00003	0.00001
40	0.00042	0.00002	0.00000	0.00000
2) Domestic supply shocks				
1	56.3[7]	34.8	79.2	43.0
4	76.2	33.3	62.2	57.9
20	77.5	33.3	62.1	54.2
40	77.7	33.5	62.1	54.5
3) Domestic demand shocks				
1	33.9	43.6	12.9	36.1
4	17.9	38.4	19.9	18.2
20	16.7	38.4	19.1	17.0
40	16.5	37.0	19.1	16.9
4) Foreign shocks				
1	2.1	2.7	0.1	6.5
4	1.1	2.3	0.2	8.0
20	1.3	2.3	0.3	8.6
40	1.4	2.3	0.3	8.5
5) Other shocks				

[7] These numbers indicate percentage of variable variance explained by given bundle of shocks over a given horizon

1	**7.6**	**19.0**	**7.7**	**14.5**
4	**4.8**	**26.0**	**17.6**	**15.9**
20	**4.5**	**26.0**	**18.5**	**20.2**
40	**4.4**	**27.2**	**18.5**	**20.1**

Source: *Own calculations.*

All domestic fundamentals, notably the GDP growth rate and domestic inflation, are mostly affected by domestic supply shocks and this effect persists over the whole forecast horizon. These shocks account for almost 80% of the volatility in GDP growth rate and more than 60% of the volatility in domestic inflation.

Domestic demand shocks are also very important factor explaining large part of variability in domestic fundamentals. They account for more than 40% of volatility in CPI inflation.

Other shocks play also significant role (and among them the inflation target shock much more than risk premium one[8]) in explaining the volatility of domestic variables – more than 20% of CPI inflation and short term interest rate.

Foreign shocks explain only negligible part of volatility of domestic variables (except for short term interest rate) in the model. This finding is not very plausible and seems to be at odds with data. Justiniano and Preston, (2010) argue that small open economy DSGE models fail to capture the impact of foreign shocks on domestic economy.

4.2 LOSS FUNCTION

Findings of the previous paragraph show that country specific shocks dominate foreign ones and this indicates that it might be costly to loose independent monetary policy. In order to compare effectiveness of a monetary policy when economy is exposed to various shocks, we can employ an approach based on a loss function. The assumption is that by implementing monetary policy central bank pursues its main objective(s) that can be pure inflation targeting, inflation targeting combined with elimination of output gap or other types of

[8] Not seen in the table but available in author's calculations

objectives. This can be formalized (see e.g. Svensson, 1997), as the monetary authority's effort to minimize intertemporal loss function

$$L_t = E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[\left(\pi_\tau - \pi^* \right)^2 + \lambda y_t^2 \right] \quad (1)$$

where $\pi_\tau - \pi^*$ is deviation of inflation from long-term inflation target, λ is the weight on output gap y_t. That is, monetary authority tries to minimize expected stream of discounted squared deviations of inflation and output from inflation target and potential output level, respectively.

Loss function may include other factors that might be important objectives for monetary authority, e.g. the difference of the short term interest rate, as in Adolfson et al., (2008) or the difference of the exchange rate. The weights on these factors can be calibrated or estimated.

In this article we are not going to construct an explicit ad-hoc loss function. Instead, we expose the estimated model to individual domestic and foreign shocks and compare the volatilities of main economic fundamentals under the regimes of independent and union monetary policies, respectively. As virtually all types of ad-hoc loss function contain deviations of inflation and output gap, in table 3 we report volatilities of these two domestic variables.

Table 3: Volatility of domestic output growth and inflation

		Var(y)	Var(п)				Var(y)	Var(п)
Shock to domestic:					**Shock to foreign:**			
government expenditure	**IMP**	**0.00041**	**8.540E-08**		**government expenditure**	**IMP**	**1.7E-07**	**5.49E-09**
	UMP	**0.00057**	**8.778E-07**			**UMP**	**1.22E-07**	**7.91E-09**
consumption	**IMP**	**0.00018**	**3.448E-07**		**consumption**	**IMP**	**2.11E-06**	**1.17E-08**
	UMP	**0.00038**	**3.010E-06**			**UMP**	**1.49E-06**	**3.03E-09**
production	**IMP**	**0.01938**	**0.0001639**		**production**	**IMP**	**0.00030**	**7.52E-06**
	UMP	**0.01797**	**0.0002449**			**UMP**	**0.00012**	**5.51E-06**
investment	**IMP**	**0.00186**	**2.004E-06**		**investment**	**IMP**	**1.09E-05**	**5.46E-08**
	UMP	**0.00195**	**1.206E-06**			**UMP**	**8.58E-06**	**1.07E-07**

IMP independent monetary policy

UMP union monetary policy

Shaded cells contain lower volatility

Source: *Own calculations.*

The following pattern emerges from the table. Independent monetary policy stabilizes domestic economy better than the union monetary policy, when economy is exposed to domestic shocks. The volatilities of output and CPI inflation are smaller in almost all cases (in 6 out of 8). On the other hand, union monetary policy does a better job in stabilizing domestic economy when it is exposed to foreign shocks. The volatilities of those variables are smaller in 6 cases out of 8. Thus the cost of loosing autonomous policy, though not quantified so far, tends to be higher when domestic shocks are more important than foreign ones, which seems to be the case for Slovak economy, according to 4.1.

4.3 WELFARE ANALYSIS

An important advantage of a model derived from household optimization, as the one that we have at hand, is the possibility to measure the consequences of alternative monetary policies by a utility-based welfare criterion, which is the main building block in its construction. The method that is most often used derives the loss function that represents the quadratic approximation to the level of expected utility of the representative household[9]. This loss function has the same form as in (1) but its components, namely weights and potential output, are well defined functions of the model's structural parameters. This approach, however, can be used only for a limited class of models; models that are not too complicated.

A recent advance in computational algorithms provides a numerical method that can be used for more complicated models. Utility function is approximated to the second or higher order[10] and welfare measure W_t – the infinite sum of discounted utilities defined by

$$W_t = E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} U\left(c_t, h_t\right) \tag{2}$$

is added to the state space. This direct observation of the variable W_t enables us to compare welfare effects of various shocks the model is subject to.

Now we expose the model to individual shocks, domestic and foreign, and track down welfare changes under the two alternative monetary policies. In table 4 we report welfare changes[11] caused by domestic and foreign temporary shocks.

[9] An extensive treatment of this subject can be found in Woodford, 2003
[10] Most software packages designed for handling DSGE modeling, including Dynare that we have used, are equipped with this capability
[11] Values in the table represent the total welfare gain (or loss) that domestic households obtain until economy stabilizes.

Table 4: Welfare changes to domestic and foreign shocks

Domestic shocks		SK_welfare change		Foreign shocks		SK_welfare change
government expenditure	IMP	-0.02216		government expenditure	IMP	-0.00327
	UMP	-0.02503			UMP	-0.00273
consumption	IMP	0.007163		consumption	IMP	-0.00925
	UMP	0.00073			UMP	-0.00887
production	IMP	0.104013		production	IMP	0.020115
	UMP	0.103276			UMP	0.026729
investment	IMP	0.026788		investment	IMP	-0.00393
	UMP	0.025494			UMP	-0.00235
				asymmetric technology	IMP	-0.00543
					UMP	0.013004

IMP **independent monetary policy**

UMP **union monetary policy**

Shaded cell contains better welfare outcome

Source: *Own calculations.*

For instance, when the model is exposed to domestic consumption shock the household welfare improves by 0.007 units under the independent monetary policy regime and by

0.0007 units under the union regime. Thus households are better off in the independent MP regime, when subject to domestic consumption shock.

In table 4 we see the pattern that we have already observed in table 3 that is, that domestic households fare better under the independent monetary policy regime, when economy is hit by domestic shocks. On the contrary, when economy is subject to foreign shocks union, the monetary policy secures better welfare outcome for domestic households.

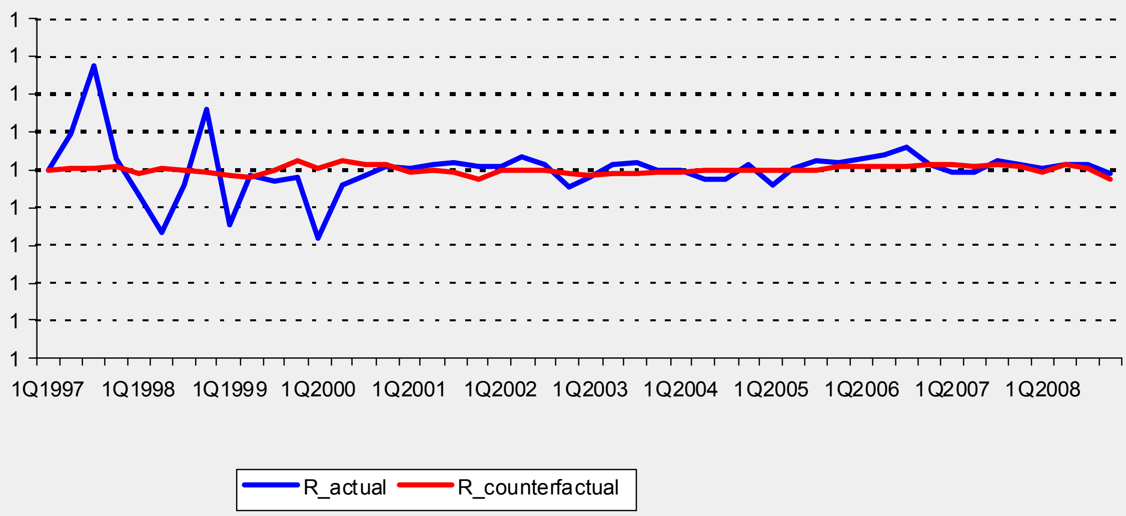
We can conclude that independent monetary policy does better job in stabilizing domestic economy when this economy is hit by domestic shocks. According to the model, the cost of loosing autonomous monetary policy can be quantified by a welfare measure, and is not very high. Union monetary policy, on the other hand, stabilizes domestic economy more efficiently and provides welfare gains[12] for domestic households, when it reacts to foreign shocks.

4.4 COUNTERFACTUAL MONETARY POLICY SHOCKS

With the help of the estimated model, the cost of monetary policy loss can be examined by conducting the following hypothetical experiment. Let us assume that Slovakia had joined monetary union in January 1999 instead of 2009. To run this experiment we simulate the model, starting from the point Slovak economy was in January 1999, by feeding it with historical exogenous shocks that have been extracted from the real data, with one notable exception. Domestic monetary shocks are replaced by such residuals that would cause the identical development of short term interest rates in both economies, i.e. how the Slovak economy would have developed if the interest rate had been set by ECB.

[12] In reality, this welfare gains can be higher than indicated in table 4 because foreign shocks might play bigger role in the volatility of the domestic indicators than is suggested in table 2. Similar study conducted by Hurnik et al. (2010) for the Czech economy indicates bigger role of foreign shocks.

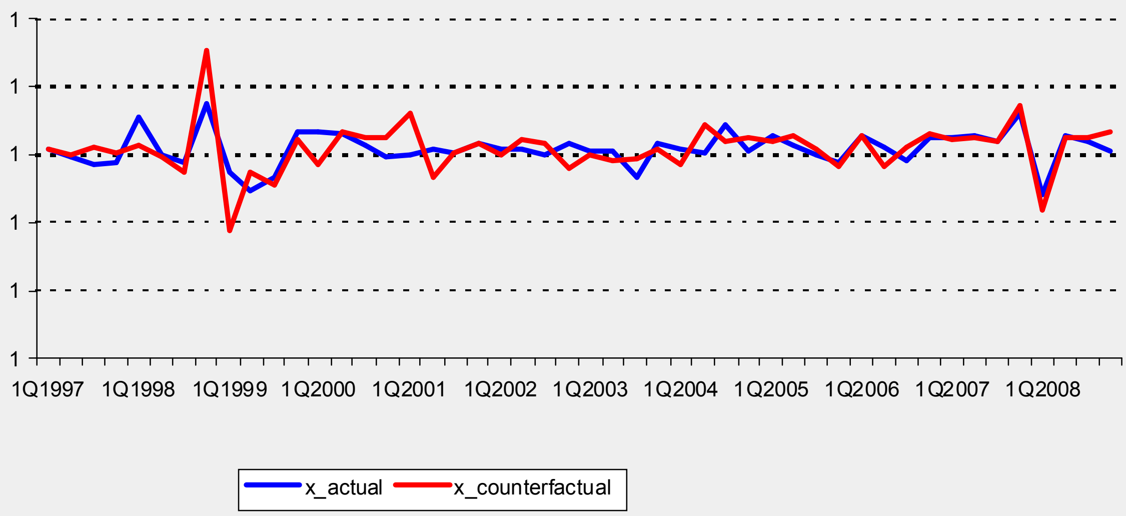
Figure 1: Interest rate



Source: *Own calculations*.

The blue line in figure 1 shows the actual short term interest rate set by the Slovak monetary authority and the red line depicts the interest rate set by the union, which is used in the counterfactual simulation. The volatility of the actual interest rate during the first 3 years is caused by the monetary policy responses to various structural changes the Slovak economy underwent during this period. As we see in figure 2, the monetary policy was quite successful in stabilizing the growth rate of domestic output (blue line). Had Slovakia joined the monetary union and adopted the union short term interest rate, the development of the growth rate of output in the first years would have been more volatile (red line). When Slovakia joined the European Union in 2004 and especially when pledged, in 2005, to adopt the common currency as soon as possible, Slovak economy started converging nominally and hence the short term interest rate set by the union became more appropriate than at the beginning of the period.

Figure 2: Growth rate of output



Source: *Own calculations*.

This counterfactual experiment supports the previous findings that the union interest rate has low stabilizing power on Slovak economy when exposed to domestic shocks, while it is reasonably effective otherwise.

5. CONCLUSION

In this article, we have quantified some costs and benefits of the euro adoption in Slovakia. Overall benefit from trade represents 0.62% of lifetime steady state consumption, which is the sum of 0.7% gain produced by higher consumption due to lower import prices caused by lower transaction costs and 0.08% loss produced by higher volatility of fundamentals.

Loss of independent monetary policy results in higher volatility of domestic fundamentals when the economy is hit by domestic shocks and resulting cost is documented by a loss function approach, by counterfactual experiment and by welfare analysis, which also provides some quantitative estimates. This cost tends to be relatively small when compared to welfare gains from trade.

It would be very presumptuous to claim that it has been beneficial for Slovakia to join the monetary union, on the basis of this article's findings only. Joining a monetary union is a considerable undertaking for an economy, accompanied by deep structural changes, leading to expected, as well as unexpected outcomes, some of them beneficial, some of them costly and many of them even impossible to quantify.

In this article, only a small fraction of them have been analyzed and this analysis finds positive welfare gains by joining the monetary union.

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APPENDIX:

THE MODEL

As mentioned above, the economy consists of two countries: a home small economy (Slovakia) and a foreign large economy (rest of the euro area). The normalised population of the overall economy is 1 with the home population equal to n and the foreign population equal to 1-n.

All real variables are expressed in terms of quantity per capita. That is why equations containing a combination of home and foreign variables (e.g. market clearing condition) have to be properly adjusted. Foreign variables are denoted by an asterisk.

In the following paragraphs, we will describe the home economy. The foreign economy has an identical structure.

FIRMS

FINAL GOODS FIRMS

There are three different types of final goods firms that combine bundles of domestically produced intermediary goods with imported intermediary goods; the firms are differentiated according to the types of non-tradable final goods they produce: private consumption goods, investment goods, and the government goods. They operate in a perfectly competitive market.

The representative firm producing private consumption goods combines a bundle of domestic intermediary goods C_t^D and a bundle of imported intermediary goods M_t^C with CES technology

$$C_t = \left[\omega_c^{\frac{1}{\mu_c}} \left(C_t^D \right)^{\frac{\mu_c - 1}{\mu_c}} + \left(1 - \omega_c \right)^{\frac{1}{\mu_c}} \left(M_t^C \right)^{\frac{\mu_c - 1}{\mu_c}} \right]^{\frac{\mu_c}{\mu_c - 1}} , \tag{1}$$

where μ_c denotes the elasticity of substitution between the domestic and imported intermediary goods and ω_c is the share of domestic intermediary goods used in production.

The bundle C_t^D comprises goods produced by domestic intermediary firms $i \in [0, n]$ while M_t^C comprises goods produced by foreign intermediary firms $i^* \in [n, 1]$

$$C_t^D = \left[\left(\frac{1}{n} \right)^{\frac{1}{\sigma_d}} \int_0^n \left(C_t^D (i) \right)^{\frac{\sigma_d - 1}{\sigma_d}} di \right]^{\frac{\sigma_d}{\sigma_d - 1}} \qquad (2)$$

$$M_t^C = \left[\left(\frac{1}{1-n} \right)^{\frac{1}{\sigma_d^*}} \int_n^1 \left(M_t^C \left(i^* \right) \right)^{\frac{\sigma_d^* - 1}{\sigma_d^*}} di^* \right]^{\frac{\sigma_d^*}{\sigma_d^* - 1}} \qquad (3)$$

σ_d, σ_d^* are elasticities of substitution between domestic and foreign intermediary goods, respectively.

Now given the composition of both bundles with prices P_t^D and P_t^{D*} respectively, firms combine these bundles in a way that minimises the total cost of production $P_t^D C_t^D + P_t^{D*} M_t^C$ subject to aggregation (1). This yields the following demand functions for the intermediary goods and price index:

$$C_t^D = \omega_c C_t \left(\frac{P_t^D}{P_t^C} \right)^{-\mu_c} \qquad (8)$$

$$M_t^C = (1-\omega_c) C_t \left(\frac{P_t^{D*} S_t}{P_t^C} \right)^{-\mu_c} \qquad (9)$$

where S_t is the nominal exchange rate expressed as an amount of home currency per unit of foreign currency $\left(SKK/€ \right)$, and the unit price of consumption goods is:

$$P_t^C = \left[\omega_c \left(P_t^D \right)^{1-\mu_c} + \left(1 - \omega_c \right) \left(S_t P_t^{D*} \right)^{1-\mu_c} \right]^{\frac{1}{1-\mu_c}} \qquad (10)$$

In the home economy, there are also representative firms producing investment goods I_t (by combining domestic bundle I_t^D with imported bundle M_t^I) and government goods G_t (produced solely out of domestic intermediary goods) , and there are similar firms in the foreign economy.

INTERMEDIATE GOODS FIRMS

There is a continuum of domestic intermediate goods firms indexed by $i \in [0,n]$, each producing differentiated goods in a monopolistically competitive market. Firm *i* rents capital $u_t K_t(i)$ from households and hires a bundle of differentiated labour services $H_t(i)$ to produce differentiated goods $X_t(i)$ with Cobb-Douglas production technology,

$$X_t(i) = \varepsilon_t^X \left(A_t H_t(i)\right)^{\alpha} \left(u_t K_t(i)\right)^{1-\alpha} \tag{11}$$

where u_t is the intensity of capital use, and bundle $H_t(i)$ combines household-specific varieties of labour in a monopolistically competitive market,

$$H_t(i) = \left[\left(\frac{1}{n}\right)^{\frac{1}{\sigma_w}} \int_0^n \left(H_t(i,j)\right)^{\frac{\sigma_w - 1}{\sigma_w}} dj \right]^{\frac{\sigma_w}{\sigma_w - 1}} \tag{12}$$

where σ_w is the elasticity of substitution between differentiated labour services, ε_t^X is a transitory technology shock, while A_t is a unit root technology shock that permanently affects labour productivity. The growth rate ε_t^A of A_t defined by $\varepsilon_t^A \equiv A_t / A_{t-1}$ is a stationary process,

$$\varepsilon_t^A = (1-\rho_A) g^A + \rho_A \varepsilon_{t-1}^A + u_t^A \tag{13}$$

where g^A is a steady-state labour productivity growth rate. It is assumed that all real variables inherited this growth rate.[13] It is also assumed that a similar labour productivity shock in foreign economy A_t^* is co-integrated with A_t. The shock $\varepsilon_t^Z \equiv A_t / A_t^*$ is then a transitory one, which measures the degree of asymmetry of the technological progress in both countries (Adolfson *et al.*, 2007).

Given the rental rate of capital Z_t^{nom} and the aggregate wage index W_t^{nom}, firm *i* wants to minimise its total cost of production $Z_t^{nom} u_t K_t(i) + W_t^{nom} H_t(i)$ subject to (11). Defining

[13] When solving the model all real variables are detrended.

$MC_t^{nom}(i)$ as a Lagrange multiplier associated with constraint (11), the first order conditions with respect to $K_t(i)$ and $H_t(i)$ lead to the following equations:

$$H_t(i) = \frac{\alpha}{1-\alpha} \frac{Z_t^{nom} u_t K_t(i)}{W_t^{nom}} \tag{14}$$

$$MC_t^{nom}(i) = \frac{1}{\varepsilon_t^X} \left(\frac{W_t^{nom}}{\alpha A_t} \right)^{\alpha} \left(\frac{Z_t^{nom}}{1-\alpha} \right)^{(1-\alpha)} \tag{15}$$

Note that the right-hand side of (15) is independent of index *i*, meaning that all firms have the same marginal cost.

As firm *i* uses a bundle $H_t(i)$ of differentiated labour services it has to decide how to optimally allocate it among household specific types *j*. Firm *i* wants to minimise labour cost $\min_{H_t(i,j)} \int_0^n W_t^{nom}(j) H_t(i,j) dj$ subject to (12), where $W_t^{nom}(j)$ is a wage rate of *j* labour type set in a monopolistically competitive market. This leads to the following demand function:

$$H_t(i,j) = \frac{1}{n} \left(\frac{W_t^{nom}(j)}{W_t^{nom}} \right)^{-\sigma_w} H_t(i) \tag{16}$$

Combining (12) with (16) we get the following expression for the aggregate nominal wage index

$$W_t^{nom} = \left[\frac{1}{n} \int_0^n W_t^{nom}(j)^{(1-\sigma_w)} dj \right]^{\frac{1}{1-\sigma_w}} \tag{17}$$

PRICE SETTING

Because intermediate firms produce differentiated goods in a monopolistically competitive market they have the market power in setting their prices. Intermediate goods are tradable and we assume that there is no price discrimination between these goods being sold domestically and abroad. Hence LOOP holds for each type of goods.

In order to accommodate inflation persistency observed in real data, we introduce sluggish price adjustment á la Calvo (Calvo, 1983). Let $P_t^D(i)$ be a price of intermediate goods produced by firm *i*. In each period, a fraction (1-τ_d) of randomly chosen firms is allowed to set their price to optimal value $P_t^{D,o}(i)$ while the remaining fraction τ_d of firms indexes their price to a combination of the inflation target and previous-period inflation,

$$P_t^D(i) = P_{t-1}^D(i)\left(\Pi_{t-1}^C\right)^{\gamma_d}\left(\overline{\Pi}^C\right)^{(1-\gamma_d)}, \tag{18}$$

where $\Pi_t^C = P_t^C / P_{t-1}^C$ is the gross inflation rate and $\overline{\Pi}^C$ is the gross steady-state inflation rate.

Firms set the same optimal price $P_t^{D,o}$ in period *t* keeping in mind that they may not be able to re-optimise in future. Thus firms select such price that maximises the present value of all future expected real profits achieved in periods when this price is just indexed but cannot be re-optimised. Such behaviour of firms leads to the following evolution of the aggregate price index P_t^D:

$$P_t^D = \left[\tau_d\left(\left(\overline{\Pi}^C\right)^{(1-\gamma_p)}\left(\Pi_{t-1}^C\right)^{\gamma_p} P_{t-1}^D\right)^{1-\sigma_d} + (1-\tau_d)\left(P_t^{D,o}\right)^{1-\sigma_d}\right]^{\frac{1}{1-\sigma_d}} \tag{21}$$

HOUSEHOLDS

There is a continuum of domestic households which, indexed[14] by $j \in [0,n]$,[15] obtain their utility from the level of consumption and from leisure. Each household *j* maximises its discounted lifetime utility at period *t* by choosing a level of real consumption $C_t(j)$, and real investment $I_t(j)$ that increases existing capital stock $K_t(j)$, the rate of capital utilisation $u_t(j)$, next period domestic bond holdings $B_{t+1}(j)$[16] and hours worked $H_t(j)$. Because each household provides differentiated labour, it has a market power in setting its wage $W_t(j)$ in a monopolistically competitive market.

The utility function has the following form:

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[14] Indexation indicates household differentiation in terms of the unique labour each household provides to the firms.
[15] The index of foreign households obtains values from [n , 1].
[16] In fact, the assumption of complete securities markets makes the existence of bonds redundant.

$$U_j = E_0 \sum_{t=0}^{\infty} \beta^t \left(\varepsilon_t^C \log\left(C_t(j) - hab \cdot C_{t-1}\right) - \frac{\left(H_t(j)\right)^{1+v}}{1+v} \right) \qquad (22)$$

where β is the discount factor, v is the inverse of elasticity of labour supply and ε_t^c is an exogenous consumption preference shock. *Hab* is a parameter of external habit formation (Abel 1990). It means that utility depends positively on the difference between contemporaneous consumption and lagged aggregate consumption. This rigidity is introduced to the model in order to improve the persistency of responses of various variables to shocks.

In optimising expression (22), household *j* faces the following budget constraint:

$$P_t^C C_t(j) + P_t^I I_t(j) + \frac{B_{t+1}(j)}{R_t} + TAX_t(j) \leq$$
$$W_t^{nom}(j) H_t(j) + \left[Z_t^{nom} u_t(j) - \Gamma^u\left(u_t(j)\right) P_t^I \right] K_t(j) + B_t(j) + d_t(j) + TR_t(j) \qquad (23)$$

where P_t^C and P_t^I are the prices of a unit of consumption and the investment goods respectively, $W_t^{nom}(j)$ is the nominal wage rate of household *j*, Z_t^{nom} is the rental rate of effective capital rented to firms, $\Gamma^u\left(u_t(j)\right)$ is the cost, in units of investment goods, of setting the utilisation rate to $u_t(j)$, $d_t(j)$ are dividends paid by firms, $TR_t(j)$ are transfers from government, and $TAX_t(j)$ are taxes paid by household *j*. Thus the left-hand side of the inequality (23) represents household *j* expenditure and the right-hand side its income.

We follow (Christiano, et al, 2001) and assume that the cost of setting the capital utilisation rate to $u_t(j)$ is given by

$$\Gamma_t^u \equiv \Gamma^u\left(u_t(j)\right) = \gamma_{u,1}(u_t(j) - 1) + \frac{\gamma_{u,2}}{2}(u_t(j) - 1)^2 \qquad (24)$$

The accumulation of physical capital owned by household *j* evolves according to:

$$K_{t+1}(j) = (1-\delta) K_t(j) + \varepsilon_t^I \left(1 - \Gamma^I \left({I_t(j)} \Big/ {I_{t-1}(j)} \right) \right) I_t(j) \qquad (25)$$

where δ is the physical capital depreciation rate, ε_t^I is an investment specific technology shock and Γ^I is the adjustment cost (Christiano, et al, 2001) of converting investment into physical capital, which has the following form:

$$\Gamma_t^I \equiv \Gamma^I \left(I_t(j) \big/ I_{t-1}(j) \right) = \frac{\gamma_I}{2} \left(\frac{I_t}{I_{t-1}} - g^A \right)^2 \tag{26}$$

with $\gamma_I \geq 0$ and g^A denoting the trend growth rate of the technology process in the steady state. The positive adjustment cost gives the household an incentive to smooth investment.

The Lagrange multipliers associated with budget constraint (23) and capital law of motion (25) are, respectively, $\lambda_{1,t}(j)$ and $\lambda_{2,t}(j)$. The first-order conditions for maximising the household's lifetime utility (22) with respect to $C_t(j)$, $B_{t+1}(j)$, $K_{t+1}(j)$, $u_t(j)$ and $I_t(j)$ are the following Euler equations:

$$\lambda_{1,t} = \varepsilon_t^C \left(P_t^C \left(C_t - hab \cdot C_{t-1} \right) \right)^{-1} \tag{27}$$

$$\lambda_{1,t} = \beta R_t \lambda_{1,t+1} \tag{28}$$

$$\lambda_{2t} P_t^I = \beta \left[\lambda_{1,t+1} u_{t+1} Z_{t+1}^{nom} - \lambda_{1,t+1} \Gamma_{t+1}^u P_{t+1}^I + (1-\delta) \lambda_{2,t+1} P_{t+1}^I \right] \tag{29}$$

$$Z_t^{nom} = (\gamma_{u,1} + \gamma_{u,2}(u_t - 1)) P_t^I \tag{30}$$

$$\lambda_{1,t} = \varepsilon_t^I \lambda_{2,t} \left[1 - \Gamma_t^I + \gamma_I \frac{I_t}{I_{t-1}} \left(\frac{I_t}{I_{t-1}} - g^A \right) \right] + \beta \varepsilon_{t+1}^I \lambda_{2,t+1} \left(\frac{I_{t+1}}{I_t} \right)^2 \pi_{t+1}^I \gamma_I \left(\frac{I_{t+1}}{I_t} - g^A \right) \tag{31}$$

Since all households make identical decisions in equilibrium, the index *j* in the equations (27)-(31) has been dropped.

Household *j* supplies differentiated labour $H_t(j)$ in a monopolistically competitive market i.e. it has a certain market power in setting its wage. It can negotiate a mark-up on labour cost. In order to emulate the wage adjustment rigidity that takes place in the real economy we introduce staggered wage setting á la Calvo (1983). A randomly chosen fraction $1-\tau_w$ of households can reset their nominal contracts at period *t*, while wages of the remaining τ_w of households are adjusted to inflation according to the following indexation scheme:

$$W_t^{nom}(j) = \Pi_{t-1}^C W_{t-1}^{nom}(j) \tag{32}$$

Each household with permission to reset its wage at period *t* maximises its lifetime utility (22) subject to the following: budget constraint (23), demand for its differentiated labour (16) and the indexation scheme (18). This leads to the following expression for the aggregate real wage index W_t:

$$W_t = \left[(1-\tau_w)\left(W_t^o\right)^{(1-\sigma_w)} + \tau_w \left(\frac{\Pi_{t-1}^C}{\Pi_t^C} W_{t-1} \right)^{(1-\sigma_w)} \right]^{\frac{1}{1-\sigma_w}} \tag{34}$$

REAL EXCHANGE RATE AND THE TERMS OF TRADE

Households in both countries buy domestic riskless bonds to insure against adverse shocks. We assume that these bonds are tradable without restriction between home and foreign households (financial markets are complete) and that there exists perfect risk-sharing across countries. Then, combining (27) with a similar equation in the foreign country (Chari et al, 1998), we can derive a relationship between the real exchange rate and the marginal utilities of consumption of the consumers in the two countries:

$$\frac{S_t P_t^{C*}}{P_t^C} \equiv S_t^{real} = \kappa \frac{\varepsilon_t^{C*} U'\left(C_t^*\right)}{\varepsilon_t^C U'\left(C_t\right)} = \kappa \frac{\varepsilon_t^{C*}\left(C_t^* - hab^* \cdot C_{t-1}^*\right)^{-1}}{\varepsilon_t^C \left(C_t - hab \cdot C_{t-1}\right)^{-1}} \tag{35}$$

Knowing the composition of consumption baskets (1) and (1*) we can establish a relationship between the real exchange rate and the terms of trade $T_t = \frac{S_t P_t^{D*}}{P_t^D}$ defined as the price of foreign goods in terms of home goods:

$$S_t^{real} = \frac{\left(\omega_c^* T_t^{1-\mu_c^*} + \left(1-\omega_c^*\right)\right)^{\frac{1}{1-\mu_c^*}}}{\left(\omega_c + \left(1-\omega_c\right) T_t^{1-\mu_c}\right)^{\frac{1}{1-\mu_c}}} \tag{36}$$

Due to different shares of domestic goods in the home and foreign consumption baskets and the potential difference between elasticities μ_c and μ_c^*, the real exchange rate deviates from the purchasing power parity (PPP) rule (deviates from 1) even though the law of one price (LOOP) holds for intermediary goods.

FISCAL AND MONETARY AUTHORITY

The fiscal authority runs a balanced budget in each period. It collects lump-sum taxes TAX_t and uses the tax revenue to purchase government goods G_t and to finance social transfers to households TR_t:

$$P_t^D G_t + TR_t = TAX_t \tag{37}$$

Government expenditure G_t is assumed to be an exogenous process that evolves according to

$$G_t = \rho_g G_{t-1} + (1-\rho_g)\overline{G} + u_t^G$$

where $\overline{G}$ is a constant fraction of total output at steady state.

In order to stabilise inflation and output in the domestic economy, the monetary authority adjusts the short-term nominal interest rate R_t in each period according to Taylor rule:

$$R_t = \left(1-\rho_R\right)\overline{R} + \rho_R R_{t-1} + \left(1-\rho_R\right)\left[\phi_\pi\left(\frac{\Pi_t^C}{\overline{\Pi^C}}\right) + \phi_x\left(\frac{X_t}{\overline{X}} - 1\right)\right] + u_t^R \tag{38}$$

where R_t is the gross nominal interest rate $\overline{R}$, $\overline{\Pi^C}$ and $\overline{X}$ are steady state values of the respective variables, and u_t^R is an *i.i.d.* monetary policy shock.

The monetary authority in foreign country sets the nominal interest rate R_t^* independently using a similar Taylor rule.

MARKET CLEARING CONDITIONS

The aggregate output of home intermediary goods is:

$$X_t = \int_0^n X_t(i)\,di = C_t^D + I_t^D + G_t + \frac{1-n}{n}\left(M_t^{C*} + M_t^{I*}\right) \tag{39}$$

where M_t^{C*} and M_t^{I*} are exported bundles of home intermediary goods used for the production of foreign consumption and investment goods,[17] respectively.

The aggregate output of foreign intermediary goods is:

$$X_t^* = \int_n^1 X_t^*(i^*)\,di^* = C_t^{D*} + I_t^{D*} + G_t^* + \frac{n}{1-n}\left(M_t^C + M_t^I\right) \tag{40}$$

REGIME SWITCH

So far, we have assumed that both countries have their own currencies, with S_t being the nominal exchange rate. Every equation derived in previous paragraphs (except (3.35) and (3.36)) has its asterisked counterpart valid in the foreign country. These equations form the model of an economy comprising two countries with autonomous monetary policies.

However, this model can be easily switched to a regime with a common currency, i.e. to a currency union. As the variable S_t can be eliminated from the model (using (3.35) and (3.36)), the regime switch does not require a change in the number of variables.

In a currency union there is one monetary authority that sets interest rate for both countries according to the following Taylor rule:

$$\begin{aligned} R_t^{EMU} &= \left(1-\rho_R^{EMU}\right)\overline{R}^{EMU} + \rho_R^{EMU} R_{t-1}^{EMU} + \\ &+\left(1-\rho_R^{EMU}\right)\left[\phi_\pi^{EMU}\left(\frac{\Pi_t^{C,EMU}}{\overline{\Pi}^{C,EMU}}\right) + \phi_x^{EMU}\left(\frac{X_t^{EMU}}{\overline{X}^{EMU}} - 1\right)\right] + u_t^{R,EMU} \end{aligned} \tag{41}$$

where $\Pi_t^{C,EMU}$ and X_t^{EMU} are euro area-wide weighted averages of inflation and output, respectively.

Hence two different Taylor rules (38) and (38)* are replaced by rule (41).

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[17] X_t is expressed "per capita n", while X_t^* is expressed "per capita (1-n)". This is why there are balancing factors in (39) and (40).

Also, in a flexible exchange rate setting we have two different Euler equations giving the price of a riskless bond in each country, obtained by combination of (27), (28) and (27)*, (28)*, respectively. In a currency union, these bond prices are equal and one Euler equation becomes redundant.

As the number of variables is not changed by switching from one regime to the other, the two equations that are dropped have to be replaced. The model for the currency union is extended by the dynamic equation for the terms of trade:

$$\frac{T_t}{T_{t-1}} = \frac{\Pi_t^{D*}}{\Pi_t^{D}} \tag{42}$$

and the nominal interest rate:

$$R_t = R_t^{*} \tag{43}$$

Financial market completeness guarantees the validity of UIP, which together with (43) implies that the nominal exchange rate S_t stays constant over time.