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MUSE: Monetary Union and Slovak Economy model

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Abstract

In this paper, the Bayesian method together with the calibration approach is used to parameterise the DSGE model. We present a medium-scale two-country model. Parameters controlling the steady state of the model are calibrated in order to match the ratios of a few selected variables to their empirical counterparts. The remaining parameters are estimated via Bayesian method. Since Slovakia has been a euro area member country for only two years, the model allows switching from an autonomous monetary policy regime to a monetary union regime. This feature enables us to parameterise the model in the case of independent monetary policy and consequently to simulate the impacts of various structural shocks on the Slovak economy as a part of the monetary union. At the end of the paper, we present the impulse-response functions of the model to selected structural shocks.

JEL classification: C11, C51,
Key words: two-country model, Bayesian methods

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INTRODUCTION

In January 2009, Slovakia joined the euro area and the euro became its official currency. As a consequence, the National bank of Slovakia ceased to conduct an autonomous monetary policy, since the European Central Bank conducts monetary policy for the whole euro area, including Slovakia. In this paper, we present theoretical foundations and simulation results of a new version of the DSGE model for the Slovak economy and the rest of the euro area. A novelty in the Slovakian context is that parameters of the model have been estimated for the first time by Bayesian technique. Data on the Slovak economy and on the rest of euro area covering the period from 1997 to 2008 were used.

Within the euro area, the Slovak economy is very small, less than 1% in terms of both GDP and population. Moreover, the foreign sector plays a very important role for the Slovak economy, with the export to GDP ratio standing at more than 80%. The most usual way of including the foreign sector is through a small open economy framework. In such a setting, the transmission of shocks is a one-way road – shocks from the large economy affect the small one, but shocks originating in the small country have no impact on the large one. The foreign sector is represented by a few exogenous variables and only shocks of these variables can be transmitted to the domestic economy. This approach has been used in a DSGE model for the Slovak economy by Zeman and Senaj (2009).

The National Bank of Slovakia, as a member of the Eurosystem, participates in policy discussions covering the entire euro area. While its main objective is still to evaluate the effects of different policies and impacts of shocks on the Slovak economy, the Slovak central bank is now more interested in the evaluation of these effects on the whole euro area. This motivation leads us to develop a two-country model in which countries form a monetary union. The Slovak economy represents one country and the rest of the euro area represents the other country. Such a model allows us to analyze various scenarios relating to both regions in a unified framework.

There are several papers devoted to multi–country models. For example, Obstfeld and Rogoff (1995) develop a two-country model based on monopolistic competition and sticky nominal prices. Pytlarczyk (2005) presents a two-country DSGE model with one country representing the German economy and the other one the rest of euro area. The structure of both economies is symmetrical and both countries form a monetary union. This setup enables an examination of how domestic as well as foreign shocks are transmitted in both regions and of their relative impact on both economies. A similar setup was used in the model of the Austrian economy developed by Breuss and Rabitsch, (2008) and in the model at the Banco de Espana by Andres et al., (2006) who augmented it with the housing and durable goods sectors. Large-scale models used in the IMF and the ECB are worth mentioning as well. The Global Economy Model (GEM) prepared by the IMF Research Department was published in 2004. The ECB staff regularly use the New Area Wide Model (NAWM – Christoffel, Coenen, Warne 2008) in the Macroeconomic Projection Exercise. Based on its predecessor, the Area Wide Model, the NAWN is a micro-founded open-economy model of the euro area. It also underpins the Euro Area and GLobal Economy model (EAGLE) developed by Gomes, Jacquinot, Pisani (2010) – which is a four-country model of the euro area and the world economy and is intended to be used for policy analysis of economic relationships across regions of the euro area and between euro area countries and the world economy.

As a benchmark, we chose the Pytlarczyk (2005) model. Our departures from the benchmark model include a different form of the investment adjustment cost function and the use of a general CES function instead of a Cobb-Douglas function to bundle differentiated intermediate goods. Furthermore, we have incorporated flexible utilisation of capital in the production process. In our framework, the euro area only trades with Slovakia, which means that the euro area region in the model represents almost a closed economy.

A complication of the modeling strategy is that we do not have a sample of data from the monetary union regime on which to estimate the model. Pytlarczyk (2005) framework has the advantage that we can easily switch between the autonomous monetary policy regime and monetary union regime. Therefore, we can use the data from the period prior to the euro adoption to estimate the model parameters. In DSGE models, it is assumed that a change in policy does not result in a change in parameter values, and thus we can switch the model to the monetary union regime.

The remainder of the paper is organised as follows. Section 1 describes the main features of respective sectors of the model, and Section 2 presents the theoretical model. In section 3, we discuss the calibration process of parameters effecting the steady state and describe data used for Bayesian estimation of the remaining parameters. Section 4 presents the impulse response functions of the model to government spending, technology and monetary shocks, respectively. Our conclusion is presented in Section 5.

1 MAIN FEATURES

The model consists of one closed economy (the euro area) that comprises two countries: home (Slovakia) and foreign (rest of the euro area). There are two regimes under which the model can operate. One regime, with autonomous currencies (including a flexible exchange rate) and autonomous monetary policies in each country, simulates the situation before Slovakia joined euro area in 2009. The other regime, with a common currency and one central bank (the ECB) that conducts monetary policy for the whole euro area, describes the situation after Slovakia joined EMU.

In each country, there are the following agents: firms (two types), households and governments.

Intermediate good firms use labour and capital to produce differentiated goods. Each firm has a monopoly in producing a specific good, hence firms operate in a monopolistic competition environment. They set prices of the produced goods by adding a markup to marginal cost. The intermediate goods are tradable.

Final good firms produce final products by mixing bundles of home and foreign intermediate goods. There is perfect competition in the final goods sector, meaning that the price of the final goods equals their marginal cost and firms earn zero profit. These firms produce three different types of non-tradable goods, namely goods used for private consumption by households, for public consumption by government, and for investment.

There is a continuum of **households,** each maximising a lifetime utility that depends on consumption and hours worked. Each household accumulates capital that rents out to firms and supplies the labour market with differentiated labour. It gives market power to workers in setting their wages. In order to improve dynamic properties, the model is amended with habit formation, investment adjustment cost and varying capacity utilisation features.

There is staggered **price setting** á la Calvo for the prices of home and foreign intermediate goods as well as staggered wage setting in the labour market. This price setting mechanism results in a certain degree of price stickiness, which together with monopolistic competition guarantees that output becomes demand driven in the short run.

Only intermediate goods can be **traded**. There is no price discrimination across markets; the same goods cost the same price in both home and foreign markets i.e. LOOP holds. This, however, does not imply that PPP holds in the short run as shares of home and foreign goods in home and foreign baskets are different in general.

Financial markets are complete, i.e. there is a worldwide market in which households in both countries can sell and buy contingent claims. This international perfect risk sharing arrangement enables us to eliminate the nominal exchange rate variable from the model. This makes switching between the model's two regimes – flexible and fixed exchange rate – very convenient.

In the flexible exchange rate regime, **monetary authorities** in both countries set the interest rate level independently, reacting to inflation and output gaps in their respective countries. In the fixed exchange rate regime, one central monetary authority sets the interest rate level for the whole region (both countries). This level of interest rates closes aggregate gaps and may not be appropriate for an individual country, particularly for the smaller one. This is the well-known problem of synchronisation and asymmetric shocks in a currency area (De Grauwe, 1997).

Fiscal authorities in both countries run a balanced budget in every period; government expenditure is financed by a non-distortionary lump-sum tax.

All real variables in both countries are assumed to have a common stochastic **trend** inherited from a non-stationary technology process. In the model, this trend is removed and these variables are assumed to be stationary.

It is also assumed that price deflators (expressed in terms of one currency) in both countries have a common trend. This makes both the real exchange rate and terms of trade variables stationary.

Hence the model consists of a set of non-linear equations where all variables are stationary. It is solved in its original form without prior linearisation (as is usually the case) using the DYNARE software package .

2 THE MODEL

As mentioned above, the economy consists of two countries: a home small economy (Slovakia) and a foreign large economy (rest of the euro area). The normalised population of the overall economy is 1 with the home population equal to n and the foreign population equal to 1-n.

All real variables are expressed in terms of quantity per head. That is why equations containing a combination of home and foreign variables (e.g. market clearing condition) have to be properly adjusted. Foreign variables are denoted by an asterisk.

In the next paragraphs, we will describe the home economy. The foreign economy has an identical structure.

2.1 FIRMS

2.1.1 FINAL GOOD FIRMS

There are three different types of final good firms that combine bundles of domestically produced intermediate goods with imported intermediate goods; the firms are differentiated according to the types of non-tradable final good they produce: a private consumption good, an investment good, and a government good. They operate in a perfectly competitive market.

The representative firm producing a private consumption good combines a bundle of domestic intermediate goods C_t^D and a bundle of imported intermediate goods M_t^C with CES technology

$$C_t = \left[\omega_c^{\frac{1}{\mu_c}} \left(C_t^D \right)^{\frac{\mu_c - 1}{\mu_c}} + \left(1 - \omega_c \right)^{\frac{1}{\mu_c}} \left(M_t^C \right)^{\frac{\mu_c - 1}{\mu_c}} \right]^{\frac{\mu_c}{\mu_c - 1}} , \qquad (1)$$

where μ_c denotes the elasticity of substitution between domestic and imported intermediate goods and ω_c is the share of domestic intermediate goods used in production.

The bundle C_t^D comprises goods produced by domestic intermediate firms $i \in [0, n]$ while M_t^C comprises goods produced by foreign intermediate firms $i^* \in [n, 1]$

$$C_t^D = \left[\left(\frac{1}{n} \right)^{\frac{1}{\sigma_d}} \int_0^n \left(C_t^D(i) \right)^{\frac{\sigma_d - 1}{\sigma_d}} di \right]^{\frac{\sigma_d}{\sigma_d - 1}} \tag{2}$$

$$M_t^C = \left[\left(\frac{1}{1-n} \right)^{\frac{1}{\sigma_d^*}} \int_n^1 \left(M_t^C(i^*) \right)^{\frac{\sigma_d^* - 1}{\sigma_d^*}} di^* \right]^{\frac{\sigma_d^*}{\sigma_d^* - 1}}, \tag{3}$$

where σ_d, σ_d^* are elasticities of substitution between domestic and foreign intermediate goods, respectively.

With prices of domestic and foreign intermediate goods being $P_t^D(i)$ and $P_t^{D*}(i^*)$, firms choose such composition of these bundles that minimises their cost. This gives the following demand functions for goods i and i*, respectively:

$$C_t^D(i) = \frac{1}{n} C_t^D \left(\frac{P_t^D(i)}{P_t^D} \right)^{-\sigma_d} \tag{4}$$

$$M_t^C(i^*) = \frac{1}{1-n} M_t^C \left(\frac{P_t^{D*}(i^*)}{P_t^{D*}} \right)^{-\sigma_d^*} \tag{5}$$

where

$$P_t^D = \left[\frac{1}{n} \int_0^n P_t^D(i)^{1-\sigma_d} di \right]^{\frac{1}{1-\sigma_d}} \tag{6}$$

$$P_t^{D*} = \left[\frac{1}{1-n} \int_n^1 P_t^{D*}(i^*)^{1-\sigma_d^*} di^* \right]^{\frac{1}{1-\sigma_d^*}} \tag{7}$$

Now given the composition of both bundles with prices P_t^D and P_t^{D*} respectively, firms combine these bundles in a way that minimises the total cost of production $P_t^D C_t^D + P_t^{D*} M_t^C$ subject to aggregation (1). This yields the following demand functions for the intermediate goods and price index:

$$C_t^D = \omega_c C_t \left(\frac{P_t^D}{P_t^C} \right)^{-\mu_c} \tag{8}$$

$$M_t^C = (1-\omega_c) C_t \left(\frac{P_t^{D*} S_t}{P_t^C} \right)^{-\mu_c} \tag{9}$$

where S_t is the nominal exchange rate expressed as an amount of home currency per unit of foreign currency $(Sk/€)$, and the price of a unit of consumption good is:

$$P_t^C = \left[\omega_c \left(P_t^D \right)^{1-\mu_c} + (1-\omega_c) \left(S_t P_t^{D*} \right)^{1-\mu_c} \right]^{\frac{1}{1-\mu_c}} \tag{10}$$

In the home economy, there are also representative firms producing investment good I_t (by combining domestic bundle I_t^D with imported bundle M_t^I) and government good G_t (produced solely out of domestic intermediate goods) , and there are similar firms in the foreign economy.

2.1.2 INTERMEDIATE GOOD FIRMS

There is a continuum of domestic intermediate good firms indexed by $i \in [0,n]$, each producing a differentiated good in a monopolistically competitive market. Firm *i* rents capital $u_t K_t(i)$ from households and hires a bundle of differentiated labour services $H_t(i)$ to produce a differentiated good $X_t(i)$ with Cobb-Douglas production technology:

$$X_t(i) = \varepsilon_t^X \left(A_t H_t(i) \right)^{\alpha} \left(u_t K_t(i) \right)^{1-\alpha}, \tag{11}$$

where u_t is the intensity of capital use, and bundle $H_t(i)$ combines household-specific varieties of labour in a monopolistically competitive market

$$H_t(i) = \left[\left(\frac{1}{n} \right)^{\frac{1}{\sigma_W}} \int_0^n \left(H_t(i,j) \right)^{\frac{\sigma_W - 1}{\sigma_W}} dj \right]^{\frac{\sigma_W}{\sigma_W - 1}}, \tag{12}$$

where σ_W is the elasticity of substitution between differentiated labour services, ε_t^X is a transitory technology shock while A_t is a unit root technology shock that permanently affects labour productivity. The growth rate ε_t^A of A_t defined by $\varepsilon_t^A \equiv A_t / A_{t-1}$ is a stationary process

$$\varepsilon_t^A = (1 - \rho_A) g^A + \rho_A \varepsilon_{t-1}^A + u_t^A \tag{13}$$

where g^A is a steady-state labour productivity growth rate. It is assumed that all real variables inherited this growth rate.[5] It is also assumed that a similar labour productivity shock in foreign economy A_t^* is co-integrated with A_t. The shock $\varepsilon_t^Z \equiv A_t / A_t^*$ is then a transitory one which measures the degree of asymmetry in the technological progress in both countries (Adolfson *et al.*, 2007).

Given the rental rate of capital Z_t^{nom} and the aggregate wage index W_t^{nom}, firm *i* wants to minimise its total cost of production $Z_t^{nom} u_t K_t(i) + W_t^{nom} H_t(i)$ subject to (11). Defining $MC_t^{nom}(i)$ as a Lagrange multiplier associated with constraint (11), the first order conditions with respect to $K_t(i)$ and $H_t(i)$ lead to the following equations:

$$H_t(i) = \frac{\alpha}{1-\alpha} \frac{Z_t^{nom} u_t K_t(i)}{W_t^{nom}} \tag{14}$$

$$MC_t^{nom}(i) = \frac{1}{\varepsilon_t^X} \left(\frac{W_t^{nom}}{\alpha A_t} \right)^{\alpha} \left(\frac{Z_t^{nom}}{1-\alpha} \right)^{(1-\alpha)}. \tag{15}$$

Note that the right-hand side of (15) is independent of index *i*, meaning that all firms have the same marginal cost.

[5] When solving the model all real variables are detrended.

As firm *i* uses a bundle $H_t(i)$ of differentiated labour services it has to decide how to optimally allocate it among household specific types *j*. Firm *i* wants to minimise labour cost $\min_{H_t(i,j)} \int_0^n W_t^{nom}(j) H_t(i,j) dj$ subject to (12), where $W_t^{nom}(j)$ is a wage rate of *j* labour type set in a monopolistically competitive market. This leads to the following demand function:

$$H_t(i,j) = \frac{1}{n}\left(\frac{W_t^{nom}(j)}{W_t^{nom}}\right)^{-\sigma_w} H_t(i) \tag{16}$$

Combining (12) with (16) we get the following expression for the aggregate nominal wage index

$$W_t^{nom} = \left[\frac{1}{n}\int_0^n W_t^{nom}(j)^{(1-\sigma_w)} dj\right]^{\frac{1}{1-\sigma_w}} \tag{17}$$

2.1.3 PRICE SETTING

Because intermediate firms produce differentiated goods in a monopolistically competitive market they have market power in setting their prices. Intermediate goods are tradable and we assume that there is no price discrimination between these goods being sold domestically and abroad. Hence LOOP holds for each good.

In order to accommodate inflation persistency observed in real data, we introduce sluggish price adjustment á la Calvo (Calvo, 1983). Let $P_t^D(i)$ be a price of intermediate good produced by firm *i*. In each period, a fraction (1-τ_d) of randomly chosen firms is allowed to set their price to optimal value $P_t^{D,o}(i)$ while the remaining fraction τ_d of firms indexes their price to a combination of the inflation target and previous-period inflation

$$P_t^D(i) = P_{t-1}^D(i)\left(\Pi_{t-1}^C\right)^{\gamma_d}\left(\overline{\Pi}^C\right)^{(1-\gamma_d)}, \tag{18}$$

where $\Pi_t^C = P_t^C / P_{t-1}^C$ is the gross inflation rate and $\overline{\Pi}^C$ is the gross steady-state inflation rate.

Firms set the same optimal price $P_t^{D,o}$ in period *t* keeping in mind that they may not be able to re-optimise in future. Thus firms select such price that maximises the present value of all future expected real profits achieved in periods when this price is just indexed but cannot be re-optimised:

$$\max_{P_t^d(i)} \sum_{k=0}^{\infty} \left(\beta\tau_d\right)^k E_t \lambda_{t,k} \left(\frac{\left(\overline{\Pi}^C\right)^{k(1-\gamma_p)} \left(\frac{P_{t+k-1}^C}{P_{t-1}^C}\right)^{\gamma_p} \left(P_t^d(i)\right)}{P_{t+k}^C} - MC_{t+k} \right) \left(\frac{\left(\overline{\Pi}^C\right)^{k(1-\gamma_p)} \left(\frac{P_{t+k-1}^C}{P_{t-1}^C}\right)^{\gamma_p} \left(P_t^d(i)\right)}{P_{t+k}^C} \right)^{-\sigma_d} X_{t+k}$$

(19)

where $\lambda_{t,k} = \left(\frac{\partial U}{\partial C}\right)_{t+k} \Big/ \left(\frac{\partial U}{\partial C}\right)_t$ is a stochastic discount factor that can be obtained from (27) and MC_t is the real marginal cost defined by $MC_t = MC_t^{nom} \big/ P_t^C$.

The solution of the maximisation problem (19) gives the optimal price:

$$P_t^{d,o} = \frac{\sigma_d}{\sigma_d - 1} \frac{\sum_{k=0}^{\infty} \left(\beta\tau_d \left(\overline{\Pi}^C\right)^{-\sigma_d(1-\gamma_p)}\right)^k E_t \left(\lambda_{t,k} \left(\frac{P_{t+k-1}^C}{P_{t-1}^C P_{t+k}^D}\right)^{-\gamma_p \sigma_d} X_{t+k} MC_{t+k} \right)}{\sum_{k=0}^{\infty} \left(\beta\tau_d \left(\overline{\Pi}^C\right)^{(1-\sigma_d)(1-\gamma_p)}\right)^k E_t \left(\lambda_{t,k} \left(\frac{P_{t+k-1}^C}{P_{t-1}^C}\right)^{\gamma_p(1-\sigma_d)} \frac{\left(P_{t+k}^D\right)^{\sigma_d}}{P_{t+k}^C} X_{t+k} \right)}$$

(20)

According to the equation (20) firms set their optimal price as a markup $\frac{\sigma_d}{\sigma_d - 1}$ over a weighted average of expected future marginal costs MC_{t+k}.

By plugging optimal price (20), and indexation (18) into equation (6), we get the equation of the aggregate price index P_t^D evolution:

$$P_t^D = \left[\tau_d \left(\left(\overline{\Pi^C} \right)^{(1-\gamma_p)} \left(\Pi_{t-1}^C \right)^{\gamma_p} P_{t-1}^D \right)^{1-\sigma_d} + (1-\tau_d)\left(P_t^{D,o} \right)^{1-\sigma_d} \right]^{\frac{1}{1-\sigma_d}} \tag{21}$$

2.2 HOUSEHOLDS

There is a continuum of domestic households which, indexed[6] by $j \in [0,n]$,[7] obtain their utility from the level of consumption and from leisure. Each household *j* maximises its discounted lifetime utility at period *t* by choosing a level of real consumption $C_t(j)$, and real investment $I_t(j)$ that increases existing capital stock $K_t(j)$, the rate of capital utilisation $u_t(j)$, next period domestic bond holdings $B_{t+1}(j)$[8] and hours worked $H_t(j)$. Because each household provides differentiated labour, it has a market power in setting its wage $W_t(j)$ in a monopolistically competitive market.

The utility function has the following form:

$$U_j = E_0 \sum_{t=0}^{\infty} \beta^t \left(\varepsilon_t^C \log\left(C_t(j) - hab \cdot C_{t-1}\right) - \frac{\left(H_t(j)\right)^{1+\nu}}{1+\nu} \right), \tag{22}$$

where β is the discount factor, ν is the inverse of elasticity of labour supply and ε_t^c is an exogenous consumption preference shock. *Hab* is a parameter of external habit formation (Abel 1990). It means that utility depends positively on the difference between contemporaneous consumption and lagged aggregate consumption. This rigidity is introduced to the model in order to improve the persistency of responses of various variables to shocks.

In optimising expression (22), household *j* faces the following budget constraint:

$$P_t^C C_t(j) + P_t^I I_t(j) + \frac{B_{t+1}(j)}{R_t} + TAX_t(j) \leq$$
$$W_t^{nom}(j) H_t(j) + \left[Z_t^{nom} u_t(j) - \Gamma^u\left(u_t(j)\right) P_t^I \right] K_t(j) + B_t(j) + d_t(j) + TR_t(j) \tag{23}$$

[6] Indexation indicates household differentiation in terms of the unique labour each household provides to firms.
[7] The index of foreign households obtains values from [n , 1].
[8] In fact, the assumption of complete asset markets makes the existence of bonds redundant.

where P_t^C and P_t^I are the prices of, respectively, a unit of consumption and the investment good, $W_t^{nom}(j)$ is the nominal wage rate of household *j*, Z_t^{nom} is the rental rate of effective capital rented to firms, $\Gamma^u(u_t(j))$ is the cost, in units of investment goods, of setting the utilisation rate to $u_t(j)$, $d_t(j)$ are dividends paid by firms, $TR_t(j)$ are transfers from government, and $TAX_t(j)$ are taxes paid by household *j*. Thus the left-hand side of the inequality (23) represents household *j* expenditure and the right-hand side its income.

We follow (Christiano, et al, 2001) and assume that the cost of setting the capital utilisation rate to $u_t(j)$ is given by

$$\Gamma_t^u \equiv \Gamma^u(u_t(j)) = \gamma_{u,1}(u_t(j)-1) + \frac{\gamma_{u,2}}{2}(u_t(j)-1)^2 \tag{24}$$

The accumulation of physical capital owned by household *j* evolves according to:

$$K_{t+1}(j) = (1-\delta)K_t(j) + \varepsilon_t^I\left(1-\Gamma^I\left(I_t(j)\middle/I_{t-1}(j)\right)\right)I_t(j) \tag{25}$$

where δ is the physical capital depreciation rate, ε_t^I is an investment specific technology shock and Γ^I is the adjustment cost (Christiano, et al, 2001) of converting investment into physical capital, which has the following form:

$$\Gamma_t^I \equiv \Gamma^I\left(I_t(j)\middle/I_{t-1}(j)\right) = \frac{\gamma_I}{2}\left(\frac{I_t}{I_{t-1}} - g^A\right)^2 \tag{26}$$

with $\gamma_I \geq 0$ and g^A denoting the trend growth rate of the technology process in the steady state. The positive adjustment cost gives the household an incentive to smooth investment.

The Lagrange multipliers associated with budget constraint (23) and capital law of motion (25) are, respectively, $\lambda_{1,t}(j)$ and $\lambda_{2,t}(j)$. The first-order conditions for maximising the household's lifetime utility (22) with respect to $C_t(j)$, $B_{t+1}(j)$, $K_{t+1}(j)$, $u_t(j)$ and $I_t(j)$ are the following Euler equations:

$$\lambda_{1,t} = \varepsilon_t^C\left(P_t^C\left(C_t - hab \cdot C_{t-1}\right)\right)^{-1} \tag{27}$$

$$\lambda_{1,t} = \beta R_t \lambda_{1,t+1} \tag{28}$$

$$\lambda_{2t} P_t^I = \beta \left[\lambda_{1,t+1} u_{t+1} Z_{t+1}^{nom} - \lambda_{1,t+1} \Gamma_{t+1}^u P_{t+1}^I + (1-\delta) \lambda_{2,t+1} P_{t+1}^I \right] \tag{29}$$

$$Z_t^{nom} = (\gamma_{u,1} + \gamma_{u,2}(u_t - 1)) P_t^I \tag{30}$$

$$\lambda_{1,t} = \varepsilon_t^I \lambda_{2,t} \left[1 - \Gamma_t^I + \gamma_I \frac{I_t}{I_{t-1}} \left(\frac{I_t}{I_{t-1}} - g^A \right) \right] + \beta \varepsilon_{t+1}^I \lambda_{2,t+1} \left(\frac{I_{t+1}}{I_t} \right)^2 \pi_{t+1}^I \gamma_I \left(\frac{I_{t+1}}{I_t} - g^A \right) \tag{31}$$

Since all households make identical decisions in equilibrium, the index *j* in the equations (27)-(31) has been dropped.

Household *j* supplies differentiated labour $H_t(j)$ in a monopolistically competitive market i.e. it has a certain market power in setting its wage. It can negotiate a markup on labour cost. In order to emulate the wage adjustment rigidity that takes place in the real economy we introduce staggered wage setting á la Calvo (1983). A randomly chosen fraction $1-\tau_w$ of households can reset their nominal contracts at period *t*, while wages of the remaining τ_w of households are adjusted to inflation according to the following indexation scheme:

$$W_t^{nom}(j) = \Pi_{t-1}^C W_{t-1}^{nom}(j) \tag{32}$$

Each household with permission to reset its wage at period *t* maximises its lifetime utility (22) subject to the following: budget constraint (23), demand for its differentiated labour (16) and the indexation scheme (18). This leads to the following expression for the optimal real wage rate:

$$W_t^o(j) = \frac{\sigma_w}{\sigma_w - 1} \frac{\sum_{k=0}^{\infty} (\beta \tau_w)^k E_t \left(H_{t+k}(j)^{1+\upsilon} \right)}{\sum_{k=0}^{\infty} (\beta \tau_d)^k E_t \left(\lambda_{t,k} H_{t+k}(j) \Pi_{t+k}^C \right)} \tag{33}$$

where $W_t^o(j)$ is the real optimal wage defined by $W_t^o(j) = W_t^{nom,o}(j) / P_t^C$.

Then the aggregate real wage index W_t evolves (with expressions (32) and (33) plugged into (17)) according to:

$$W_t = \left[(1-\tau_w)\left(W_t^o\right)^{(1-\sigma_w)} + \tau_w \left(\frac{\Pi_{t-1}^C}{\Pi_t^C} W_{t-1} \right)^{(1-\sigma_w)} \right]^{\frac{1}{1-\sigma_w}} . \tag{34}$$

2.3 REAL EXCHANGE RATE AND THE TERMS OF TRADE

Households in both countries buy domestic riskless bonds to insure against adverse shocks. We assume that these bonds are tradable without restriction between home and foreign households (financial markets are complete) and thus that there exists perfect risk-sharing across countries. Then, combining (27) with a similar equation in the foreign country (Chari et al, 1998), we can derive a relationship between the real exchange rate and the marginal utilities of consumption of the consumers in the two countries:

$$\frac{S_t P_t^{C*}}{P_t^C} \equiv S_t^{real} = \kappa \frac{\varepsilon_t^{C*} U'\left(C_t^*\right)}{\varepsilon_t^C U'\left(C_t\right)} = \kappa \frac{\varepsilon_t^{C*}\left(C_t^* - hab^* \cdot C_{t-1}^*\right)^{-1}}{\varepsilon_t^C\left(C_t - hab \cdot C_{t-1}\right)^{-1}} \tag{35}$$

Knowing the composition of consumption baskets (1) and (1*) we can find a relationship between the real exchange rate and the terms of trade $T_t = \frac{S_t P_t^{D*}}{P_t^D}$ defined as the price of foreign goods in terms of home goods:

$$S_t^{real} = \frac{\left(\omega_c^* T_t^{1-\mu_c^*} + \left(1-\omega_c^*\right)\right)^{\frac{1}{1-\mu_c^*}}}{\left(\omega_c + \left(1-\omega_c\right) T_t^{1-\mu_c}\right)^{\frac{1}{1-\mu_c}}} \tag{36}$$

Due to different shares of domestic goods in the home and foreign consumption baskets and to the potential difference between elasticities μ_c and μ_c^*, the real exchange rate deviates from the purchasing power parity (PPP) rule (deviates from 1) even though the law of one price (LOOP) holds for intermediate goods.

2.4 FISCAL AND MONETARY AUTHORITY

The fiscal authority runs a balanced budget in each period. It collects lump-sum taxes TAX_t and uses the tax revenue to purchase government goods G_t and to finance social transfers to households TR_t :

$$P_t^D G_t + TR_t = TAX_t \qquad (37)$$

Government expenditure G_t is assumed to be an exogenous process that evolves according to

$$G_t = \rho_g G_{t-1} + (1-\rho_g)\overline{G} + u_t^G$$

where $\overline{G}$ is a constant fraction of total output at steady state.

In order to stabilise inflation and output in the domestic economy, the monetary authority adjusts the short-term nominal interest rate R_t in each period according to Taylor rule:

$$R_t = \left(1-\rho_R\right)\overline{R} + \rho_R R_{t-1} + \left(1-\rho_R\right)\left[\phi_\pi\left(\frac{\Pi_t^C}{\overline{\overline{\Pi^C}}}\right) + \phi_x\left(\frac{X_t}{\overline{\overline{X}}} - 1\right)\right] + u_t^R \qquad (38)$$

where R_t is the gross nominal interest rate $\overline{R}$, $\overline{\Pi^C}$ and $\overline{X}$ are steady state values of the respective variables, and u_t^R is an *i.i.d.* monetary policy shock.

The monetary authority in foreign country sets the nominal interest rate R_t^* independently using a similar Taylor rule.

2.5 MARKET CLEARING CONDITIONS

The aggregate output of home intermediate goods is:

$$X_t = \int_0^n X_t\left(i\right)di = C_t^D + I_t^D + G_t + \frac{1-n}{n}\left(M_t^{C*} + M_t^{I*}\right) \qquad (39)$$

where M_t^{C*} and M_t^{I*} are exported bundles of home intermediate goods used for the production of foreign consumption and investment goods,[9] respectively.

[9] X_t is expressed "per head of population n", while X_t^* is expressed "per head of population (1-n)". This is why there are balancing factors in (39) and (40).

The aggregate output of foreign intermediate goods is:

$$X_t^* = \int_n^1 X_t^*\left(i^*\right)di^* = C_t^{D*} + I_t^{D*} + G_t^* + \frac{n}{1-n}\left(M_t^C + M_t^I\right) \tag{40}$$

2.6 REGIME SWITCH

So far, we have assumed that both countries have their own currencies, with S_t being the nominal exchange rate. Every equation derived in previous paragraphs (except (3.35) and (3.36)) has its asterisked counterpart valid in the foreign country. These equations form the model of an economy comprising two countries with autonomous monetary policies.

However, this model can be easily switched to a regime with a common currency, i.e. to a currency union. As the variable S_t can be eliminated from the model (using (3.35) and (3.36)), the regime switch does not require a change in the number of variables.

In the currency union there is one monetary authority that sets interest rate for both countries according to the following Taylor rule:

$$\begin{aligned} R_t^{EMU} &= \left(1-\rho_R^{EMU}\right)\overline{R}^{EMU} + \rho_R^{EMU} R_{t-1}^{EMU} + \\ &+\left(1-\rho_R^{EMU}\right)\left[\phi_\pi^{EMU}\left(\frac{\Pi_t^{C,EMU}}{\overline{\overline{\Pi}}^{C,EMU}}\right) + \phi_x^{EMU}\left(\frac{X_t^{EMU}}{\overline{\overline{X}}^{EMU}} - 1\right)\right] + u_t^{R,EMU} \end{aligned} \tag{41}$$

where $\Pi_t^{C,EMU}$ and X_t^{EMU} are euro area wide weighted averages of inflation and output, respectively.

Hence two different Taylor rules (38) and (38)* are replaced with the rule (41).

Also, in a flexible exchange rate setting we have two different Euler equations giving the price of a riskless bond in each country, obtained by combination of (27), (28) and (27)*, (28)*, respectively. In the currency union, these bonds' prices are equal and one Euler equation becomes redundant.

As the number of variables is not changed by switching from one regime to the other, the two equations that are dropped have to be replaced. The model for the currency union is extended by the dynamic equation for the terms of trade:

$$\frac{T_t}{T_{t-1}} = \frac{\Pi_t^{D*}}{\Pi_t^D} \tag{42}$$

and the nominal interest rate:

$$R_t = R_t^* \tag{43}$$

Financial market completeness guarantees the validity of UIP, which together with (43) implies that the nominal exchange rate S_t stays constant over time.

3 PARAMETERISATION OF THE MODEL

The process of parameterisation of the model consists of two steps. First, we calibrate parameters that determine the non-stochastic steady state of the model. In the second step, we estimate the remaining structural parameters and standard deviations of the shocks by Bayesian method. Because steady state assumptions of the model[10] are not fully in line with the observed data, we decided to calibrate a subset of parameters instead of estimating them. Especially in the beginning of our sample, investment and government expenditure shares of output follow a downward trend. For this reason, we calibrate the steady state on a shorter period, from 2005 to 2008. The details on parameterisation come in the sections following the description of the data used in the estimation.

3.1 DATA

In the estimation, we use the following set of six variables for each country

- real GDP (X)
- real consumption (C)
- real investment (I)
- real compensation (W)
- short-term (3M) nominal interest rate (R)
- GDP deflator (p).

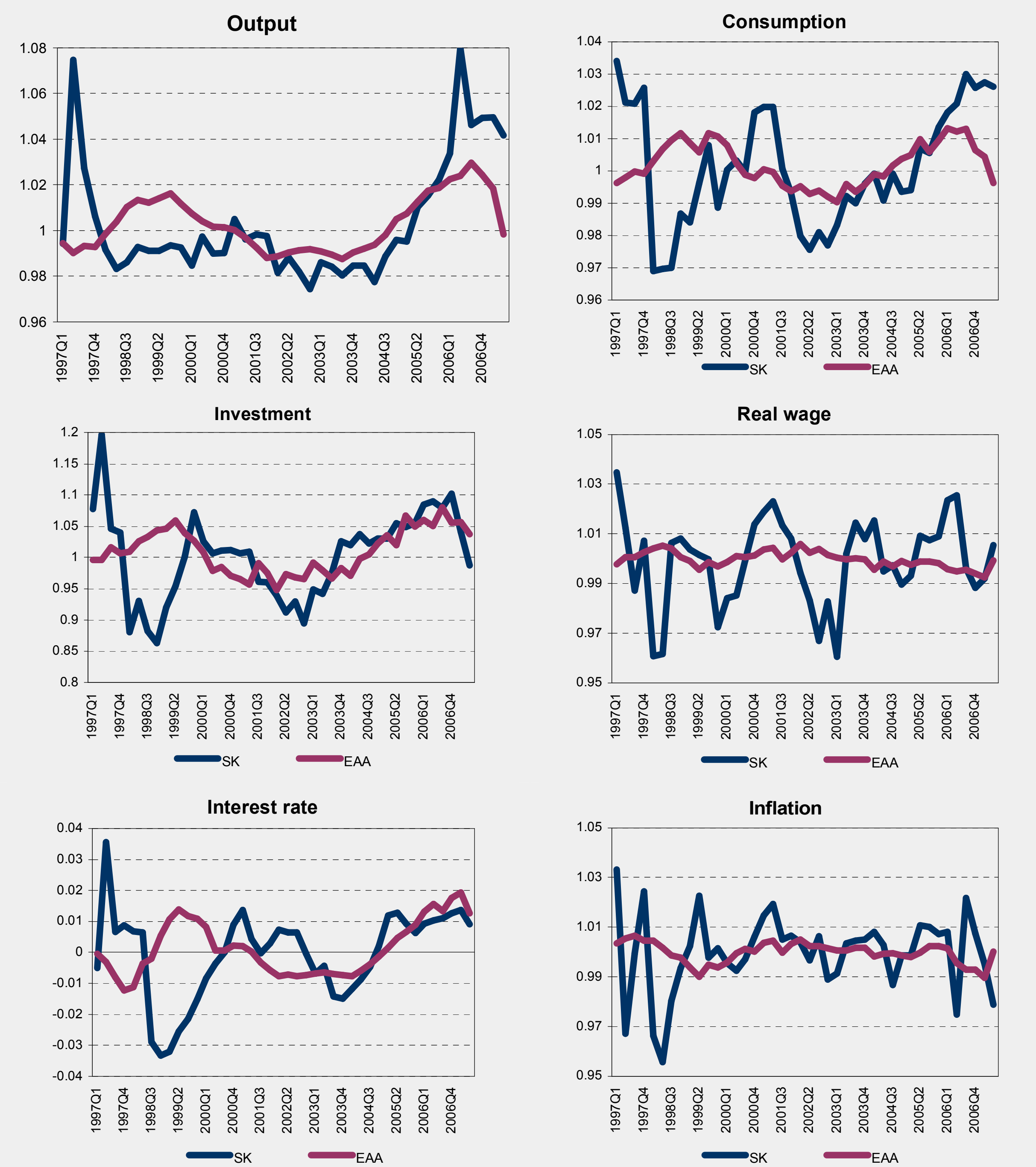
We also use government expenditures to estimate the autoregressive process of the corresponding variable in the model. All variables come from the Eurostat database, are seasonally adjusted, and are expressed in per capita terms. The length of the period in the model is one quarter. We further assume that the euro area consists of twelve countries[11] over the whole sample, although the actual composition changed several times.

The sample starts at the first quarter of 1997 and ends at the fourth quarter of 2008. With Slovakia having been a member of the euro area since the beginning of 2009, the last observation in the sample is the last quarter of autonomous monetary policy in the country. The financial crisis contaminates the last few observations in the sample. Although the model is not able to explain this episode properly, we keep these observations in the estimation because the sample is short and we want to use as many observations as possible.

We remove the trend in the data by using an HP filter with a smoothing parameter equal to 1600. The detrended data are then linked to the model variables via a state-space representation presented later in the text. Our observables are presented in Figure 1.

[10] The balanced growth incorporated in the model implies that shares of consumption, investment, government spending and net exports in output are constant.
[11] The following countries are assumed to form the euro area: Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, the Netherlands, Portugal and Spain.

Figure 1 HP filtered data



3.2 CALIBRATION OF THE COEFFICIENTS

The parameters that determine the non-stochastic steady state take values such that the ratios of a few selected model variables correspond to their empirical counterparts. In particular, we match the ratios of investment, government, and net exports to output. The following table presents the steady state properties of the model.

Table 1 Steady state ratios

	SK	EA
I/X	0.22	0.20
C/X	0.55	0.60
Im/X	0.26	0.003
ImI/X	0.10	0.001
ImC/X	0.16	0.003
G/X	0.18	0.20
TB/X	0.00	0.00
n	0.01	0.99

Since Slovakia has been a transition country, the empirical ratios that we aim to match are far from steady. For example, investment and government expenditures as shares of output in real terms tend to decline over the sample period, while net exports improve. Moreover, GDP per capita in Slovakia is well below the euro area GDP per capita, while investment share in output is higher in Slovakia. Given the model structure, these two empirical facts cannot hold in steady state. A higher saving rate means higher capital stock, which in turn means higher output per capita. Therefore, we decided to set the investment to GDP ratio in Slovakia and in the euro area to 22% and 20%, respectively. The government size is 18% in Slovakia and 20% in the euro area.

Regarding international trade flows, it is sufficient to specify the size of trade and the trade balance of one country. Naturally, we primarily focus on Slovakia. In order to obtain the specification of the trade, some necessary adjustments to the data need to be made.

In the actual data, a substantial part of imported goods is used as an input in the output production of export sectors. In the model, however, all imports are consumed domestically. There is empirical evidence that about 70% of all Slovak imports enter the production process of the exporting sectors. We assume that Slovakia exchanges its entire trade volume with the euro area. Thus we set the Slovak imports to GDP ratio at 26%. As for the trade balance, we make a simplifying assumption and set this variable at zero.

Along with the total imports of both countries, we need to specify the share of imported investment and consumption goods. Of total Slovak imports, investment goods represent

40% and consumption goods 60%. The small size of Slovakia compared to the euro area means that the euro area is more or less a closed economy. We assume that Slovakia exports substantially fewer investment goods than consumption goods. Thus, in the euro area, the investment share of imports is only 19% while the consumption share reaches 81%.

The balanced growth assumption adopted in the model requires that the two countries share the long-term technological progress. To meet this assumption we set the technological growth rate in both countries in line with Pytlarczyk (2005), at 1.6% p.a., even though the average growth rate of the Slovak GDP is considerably higher than the growth rate of the euro area GDP.

The size of Slovakia in the modelled world is 1% whereas the size of the euro area is 99%. The set of parameters that we fix symmetrically across the two regions include: labour share in production α, depreciation rate δ, discount factor β, and intratemporal elasticities of substitution between the domestic and imported bundles of investment and consumption goods μ_C, μ_I, σ_w and ν. We present the values of all calibrated parameters in Table 4 in the Appendix.

3.3 BAYESIAN ESTIMATION OF THE PARAMETERS

The Bayesian approach is widely used as an estimation tool when working with DSGE models, with such approach proposed by a clutch of recent books and papers, e.g. Canova (2007), Christoffel et. al. (2009) Smets & Wouters (2007), Almeida (2009), Schorfheide (2000).

The novelty of this paper is that we estimate selected parameters of the model by Bayesian method. To our knowledge, the parameters describing the Slovak economy have not so far been estimated via Bayesian approach.

Using this approach, we estimate 34 parameters of the model, for example, coefficients describing the monetary policy, wage and price setting, and adjustment cost. These parameters, stacked in vector θ, do not affect steady state. Moreover, the parameters calibrating the structural shocks, e.g. autoregressive coefficients and standard errors, are estimated for both countries.

Bayes' theorem tells us that posterior distribution $p\left(\theta \middle| Y^{data}\right)$ can be obtained from prior beliefs about parameter values, summarised in prior distribution $p(\theta)$, and from information on empirical data and suggested model structure, summarised in the likelihood function $p\left(Y^{data} | \theta\right)$. The mathematical representation of the Bayes' rule is as follows:

$$p\left(\theta \middle| Y^{data}\right) = \frac{p\left(Y^{data} | \theta\right) p(\theta)}{p\left(Y^{data}\right)}.$$

Since, $p\left(Y^{data}\right)$ is constant with respect to θ, then it can be rewritten as follows:

$$p\left(\theta \middle| Y^{data}\right) \propto p\left(Y^{data} \middle| \theta\right) p(\theta).$$

Posterior distribution can be evaluated for any given value of θ. But, in general, the whole distribution of $p\left(\theta \middle| Y^{data}\right)$ is unknown.

Therefore, the Metropolis-Hastings algorithm is used to approximate the posterior distribution. The M-H algorithm belongs to the group of Markov Chain Monte Carlo (MCMC) methods. The basic aim is quite straightforward – to produce a Markov chain with desired ergodic distribution, the distribution in our case being equal to $p\left(\theta \middle| Y^{data}\right)$. Consequently, after a large number of steps, the state of the chain is used as a sample from posterior distribution.

One of the shortcomings of the Bayesian approach is that the likelihood, and consequently the posteriors, are sensitive to the selection of observables. There is a rule that the number of observables has to be lower or equal to the number of structural shocks embodied in the model. The paper by Guerron-Quintana (2010) provides evidence of the sensitivity mentioned earlier. It estimates the same model on different subsets of observables in which some observable is missing. It claims that, depending on the dataset, the point estimates of habit formation range from 0.7 to 0.97. This paper concludes that point estimates are influenced more by the omission of some observables than by the choice of the shorter sample. Therefore, we use as many observables as we are allowed to.

When estimating the model we use 12 observables, thus the number of structural shocks equals the number of observables. In order to build a link between our model and the empirical data, the model was extended by the following measurement equations:

Table 1 Measurement equations

Slovakia	Euro area
$\hat{X}_t^{obs} = X_t / \bar{X}$	$\hat{X}_t^{*obs} = X_t^* / \bar{X}^*$
$\hat{C}_t^{obs} = C_t / \bar{C}$	$\hat{C}_t^{*obs} = C_t^* / \bar{C}^*$
$\hat{I}_t^{obs} = I_t / \bar{I}$	$\hat{I}_t^{*obs} = I_t^* / \bar{I}^*$
$\hat{W}_t^{obs} = W_t / \bar{W}$	$\hat{W}_t^{*obs} = W_t^* / \bar{W}^*$
$\hat{p}_t^{d,obs} = p_t^d / \bar{p}^d$	$\hat{p}_t^{d*,obs} = p_t^{*d} / \bar{p}^{*d}$
$\hat{R}_t^{obs} = R_t^4 - \bar{R}^4$	$\hat{R}_t^{*obs} = R_t^{*4} - \bar{R}^{*4}$

The left-hand side of the measurement equations stands for the observables, which are detrended using the HP filter. This means that $\hat{X}_t^{obs} = X_t^{obs} / \bar{X}^{HP}$. Combining the empirical data and the model structure with the help of a Kalman filter, the likelihood function can be computed.

According to the Bayes rule, the posterior is equal to likelihood times prior, where the priors represent additional information added to the estimation procedure. Thus, priors can be seen as the researcher belief about structural parameters. Fernández-Villaverde (2009) argues that tighter priors are the better option if the model is to be used for policy analysis. By contrast, looser priors (e.g. uniform priors) are the preferred option if the model is to be used for pure research. Loose priors let the likelihood dominate the posterior. Since our model is assumed to be used for policy simulation, we prefer tighter priors with reasonable standard deviation.

When setting the priors in this paper, four types of probability distribution were used. In the case of the parameters constrained between 0 and 1, the Beta distribution is employed. The prior for parameter of investment adjustment cost is set as a Gamma distribution. Normal distribution is used for two parameters in the Taylor rule, namely response to the output gap and response to the deviation of inflation from its steady state value. Finally, Inverse-gamma distribution is used for standard deviations of the structural shocks. The particular type of distribution, its mean, and the standard deviation for each parameter is shown in Table 4 in the Appendix.

Using a Metropolis-Hastings algorithm, we generated 350 000 draws from the posterior. This procedure was repeated with two different Markov chains. The posterior characteristic – such as median, mean, mode and confidence intervals – are reported in Table 2. In the following part, we discuss the means of the estimated parameters.

Investment is subject to a moderate adjustment cost. The estimated adjustment cost parameter in Slovakia is higher than in the euro area, as expected given the higher volatility of Slovakian investment data. Moreover, the volatility of the investment shock in Slovakia is approximately four times higher than in euro area.

In the case of price and wage adjustments, the estimated parameters for the Slovak economy are similar to those for the euro area. In Slovakia, the probability of no wage change is around 0.71, while in the euro area it is estimated at 0.78. The degree of price indexation is 0.5, which is lower than the 0.69 recorded in the euro area. Another difference is price flexibility. According to the estimated fraction of firms that are not allowed to set optimal prices in the current quarter, we can conclude that prices in Slovakia are more flexible than in the euro area. The estimated values are 0.43 and 0.79 in Slovakia and the euro area, respectively. This implies shorter average price contract duration of 1.75 quarters in Slovakia.

The estimated Taylor rule for Slovakia has a very important degree of inertia. The degree of interest rate smoothing is slightly lower than 0.9. The remaining two Taylor rule parameters have different interpretations. On one hand, there is significant sensitivity to consumer price inflation – the estimated weight is around 2. On the other hand, sensitivity to the output gap is relatively low, at 0.2.

In the case of standard deviations of four comparable structural shocks (namely preference, investment, technology, and monetary shocks), we found that they are significantly higher in Slovakia than in the euro area. The conclusion that exogenous shocks are more volatile in Slovakia is in line with the higher volatility in empirical data, as can be seen in Figure 1.

Table 2 Posterior distribution

Parameter		Mode	Mean	Median	Confidence interval	
Slovakia						
Adjustment costs						
Parameter of investment adj. cost	γ_I	4.984	4.953	4.677	1.244	8.662
Parameter of capital utilisation cost	$\gamma_{u,2}$	0.242	0.269	0.260	0.065	0.473
Wage and price setting						
Price indexation	γ_p	0.503	0.491	0.488	0.256	0.726
Probability of no price change	τ_d	0.458	0.434	0.433	0.288	0.581
Probability of no wage change	τ_w	0.764	0.710	0.715	0.488	0.933
Taylor rule						
Interest rate smoothing	ρ_R	0.888	0.875	0.878	0.830	0.921
Resp. to inflation	ϕ_π	2.151	2.083	2.074	1.391	2.775
Resp. to output growth	ϕ_x	0.253	0.219	0.217	0.010	0.428
Autoregressive coefficients						
Preference shock	ρ_C	0.580	0.530	0.533	0.319	0.740
Investment shock	ρ_I	0.139	0.146	0.135	-0.029	0.320
Covariance stationary technology shock	ρ_X	0.332	0.378	0.363	0.160	0.596
Technology growth shock	ρ_A	0.563	0.501	0.513	0.173	0.829
Asymmetric technology innovation	ρ_Z	0.711	0.709	0.709	0.674	0.744
Standard deviations						
Preference shock		0.041	0.042	0.042	0.033	0.051
Investment shock		0.255	0.256	0.243	0.079	0.434
Cov. stat. techn. shock		0.058	0.058	0.055	0.030	0.085
Monetary shock		0.005	0.006	0.006	0.004	0.007
Technology growth shock		0.004	0.004	0.004	0.002	0.007
Asymmetric technology innovation		0.018	0.018	0.018	0.014	0.021
Euro area						
Adjustment costs						
Parameter of investment adj. cost	γ_I^*	4.706	3.910	3.832	1.605	6.216
Parameter of capital utilisation cost	$\gamma_{u,2}^*$	0.011	0.019	0.015	0.005	0.033
Wage and price setting						
Price indexation	γ_p^*	0.661	0.696	0.703	0.416	0.976

Probability of no price change	τ_d^*	0.798	0.790	0.791	0.710	0.870
probability of no wage change	τ_w^*	0.792	0.783	0.789	0.627	0.940
Taylor rule						
Interest rate smoothing	ρ_R^*	0.904	0.896	0.898	0.855	0.937
Resp. to inflation	ϕ_π^*	1.670	1.652	1.652	1.453	1.852
Resp. to output growth	ϕ_x^*	0.148	0.153	0.153	0.070	0.236
Autoregressive coefficients						
Preference shock	ρ_C^*	0.754	0.691	0.693	0.487	0.894
Investment shock	ρ_I^*	0.516	0.515	0.516	0.326	0.703
Covariance stationary technology shock	ρ_X^*	0.457	0.406	0.401	0.180	0.632
Standard deviations						
Preference shock		0.018	0.019	0.019	0.015	0.023
Investment shock		0.073	0.064	0.062	0.025	0.103
Cov. stat. techn. shock		0.037	0.041	0.038	0.017	0.065
Monetary shock		0.001	0.001	0.001	0.001	0.002

Note: Confidence interval equals to mean ±2 st. dev.*

4 IMPULSE RESPONSE FUNCTIONS

In this section, we comment on the reactions of a few selected model variables to three different shocks: temporary technology shock, government spending shock and monetary policy shock. For each shock, we report the impulse response functions in both monetary regimes: the independent monetary policy regime and monetary union regime.

For the former regime, we report Bayesian IRFs while for the latter regime we base reactions on a single set of parameters that come from Bayesian estimation of the model under the independent monetary regimes.[12] We assume that the change of the monetary policy regime did not change the structural parameters of the technology and preferences.[13] The only difference is that we adopt the Taylor rule of the euro area for both regions of the model. The parameters of the euro area did not change at all.

4.1 GOVERNMENT SPENDING SHOCK

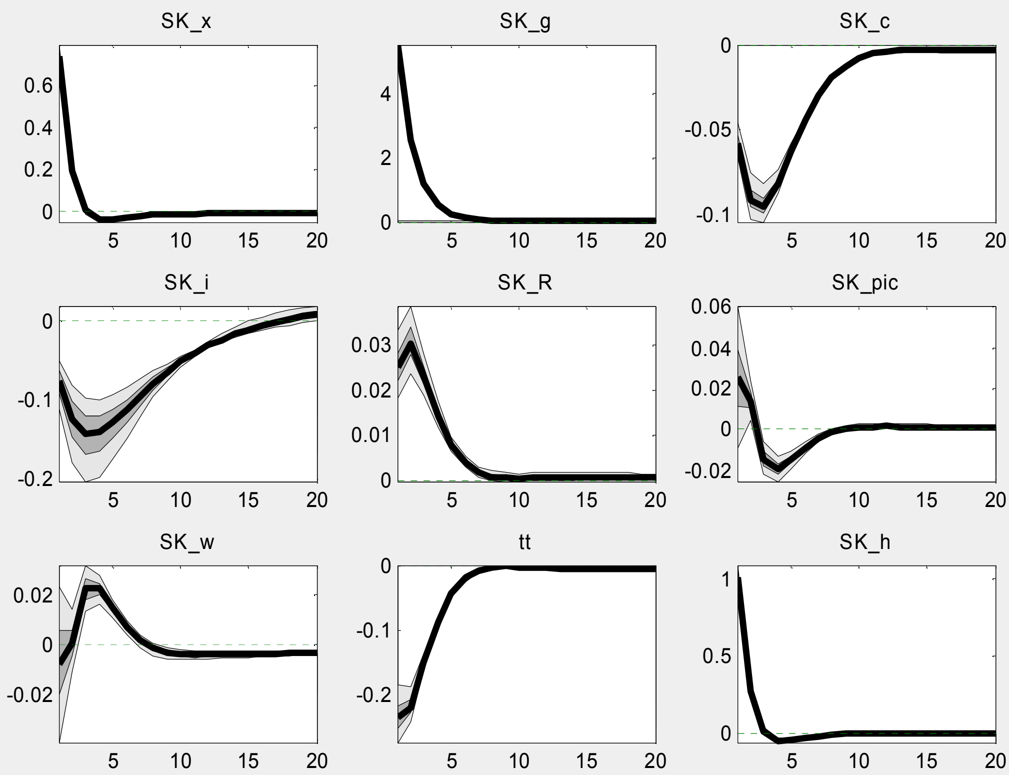
Autonomous monetary policy

This simulation represents a persistent demand shock when the initial increase in government spending is set to 1% of GDP. Thereafter, the spending remains above its steady state value for a protracted period given by the AR coefficient of the government expenditures rule.[14]

In the model, the government consumes only domestically-produced intermediate goods. A rapid increase in demand for the goods puts pressure on the marginal costs of production and factor prices. This negative wealth effect leads to a decrease in private consumption and investment. The extent of the crowding-out effect is not very large due to the open economy dimension when households are able to borrow from abroad. We see this in higher imports of both consumption and investment goods. The accompanying fall in terms of trade further mitigates the negative effects of government demand. However, this also leads to lower foreign demand and has a dampening effect on output. Another important effect in the optimising behaviour of households is the requirement that the government budget for every period has to balance. In order to finance high spending, government needs to impose higher (lump-sum) taxes, which in turn immediately reduces the disposable income of households. The implied fall in consumption makes households more willing to work and thus the fall in household expenditure does not have to be dramatic.

[12] More specifically, we use median values of the estimated parameters.
[13] There is some evidence that the deep parameters of DSGE models may not be structural in the sense of Lucas critique that these parameters are invariant to policy changes (see Fernández-Villaverde, Rubio-Ramírez, 2008). However, it is not possible to estimate the model on the common monetary regime due to the lack of data.
[14] The autoregressive coefficient of the government shock is equal to 0.46.

Figure 2 Government spending shock – autonomous monetary policy



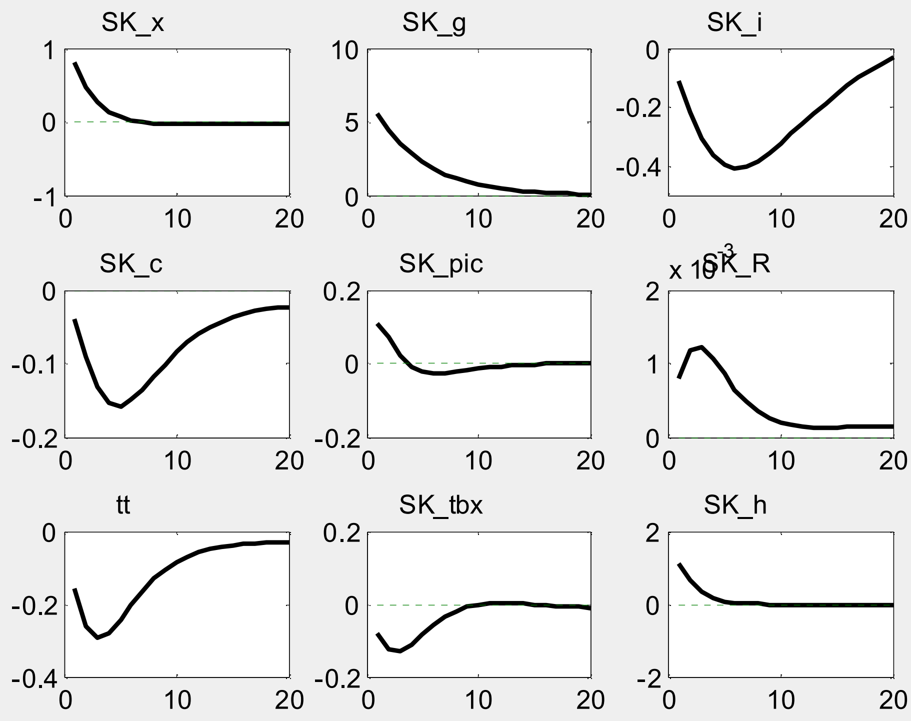
Note: Drawn at posterior mean (solid) with 40% (dark shaded) and 80% (light shaded) intervals. Vertical axes correspond to percentage deviations from the steady state. There is an exception for interest rates (R) and inflation (pic) where vertical axes correspond to percentage points deviations.

Together these effects cause output to rise by about 0.6% in the first period. Thereafter, output gradually returns to its steady state value. Consumer prices follow the rising costs of production and consumer inflation exceeds the target by 0.03 percentage point in the first period. Due to the considerable interest rate smoothing behaviour of the central bank and the dampening effect of the exchange rate, the interest rate rises only slightly.

Monetary union

When the country joins the monetary union and the exchange rate channel is shut down the crowding-out effect in investment is stronger. The exchange rate does not create any positive wealth effect, and as import growth decelerates, the worsening of the trade balance becomes less pronounced. On the other hand, there is a positive intertemporal effect due to the decline in real interest rates. Real interest rates decline because nominal rates do not change while inflation picks up. This effect makes households willing to consume more today and compensates them for the negative wealth effect. Inflation in consumer prices is, however, higher. The effect on consumption and labour is about the same.

Figure 3 Government spending shock – common monetary policy



4.2 TECHNOLOGY SHOCK

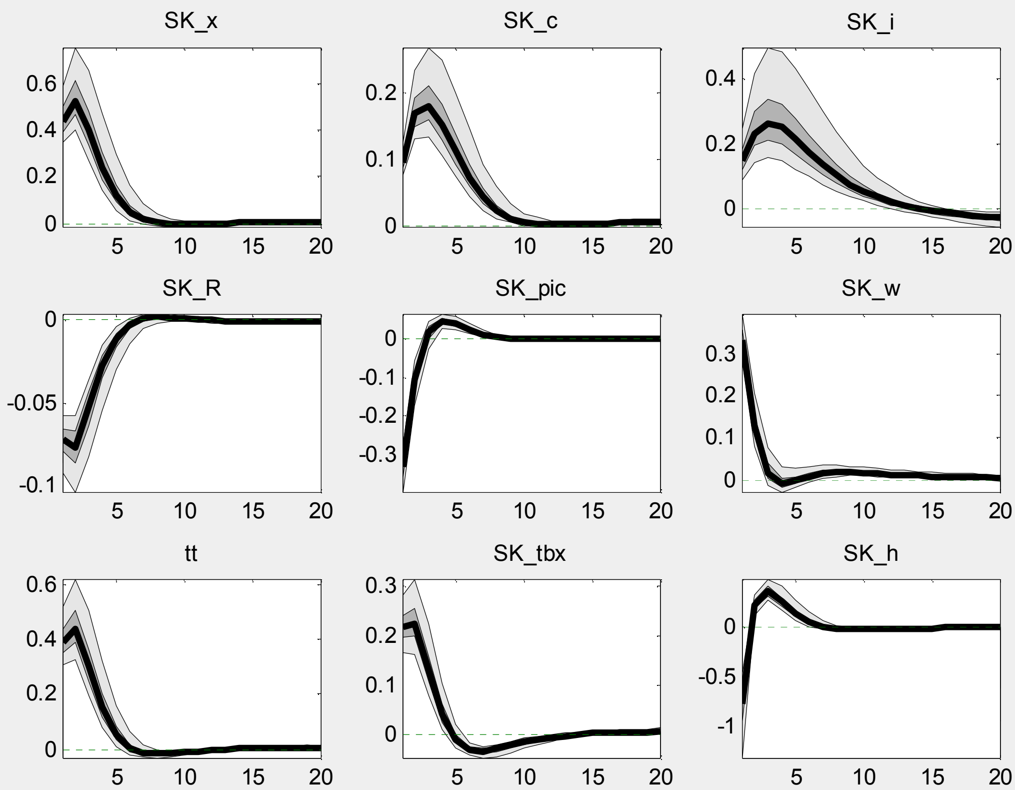
Autonomous monetary policy

The technology shock is implemented as a temporary, yet persistent[15], improvement in the production technology. The size of the initial shock is 1%.

After the positive shock to the factor of productivity, marginal costs fall and households demand more consumption, investment, and are less willing to work. Nominal wages do not change significantly while real wages rise as a result of lower inflation. Due to the expected higher lifetime income, consumers tend to immediately increase their consumption, and the higher return on capital motivates them to invest more. Higher consumption makes leisure less expensive, hence households decrease their supply of labour initially. However, as the positive impact of technology is only temporary, households increase their labour supply when the shock fades away. In fact, the labour supply even exceeds the steady state level of labour for a few periods. The positive wealth effect is also reinforced by the positive substitution effect, since prices decrease while interest rates do not adjust enough and real rates thus decline. Households therefore prefer early consumption. Lower production prices improve the competitiveness of the economy, which appears as an improvement in the terms of trade and leads domestic consumers to switch away from imported goods and replace them with domestic production. At the same time, foreign consumers demand more imports for the same reason.

[15] The autoregressive coefficient of the shock process is equal to 0.36

Figure 4 Technology shock – autonomous monetary policy



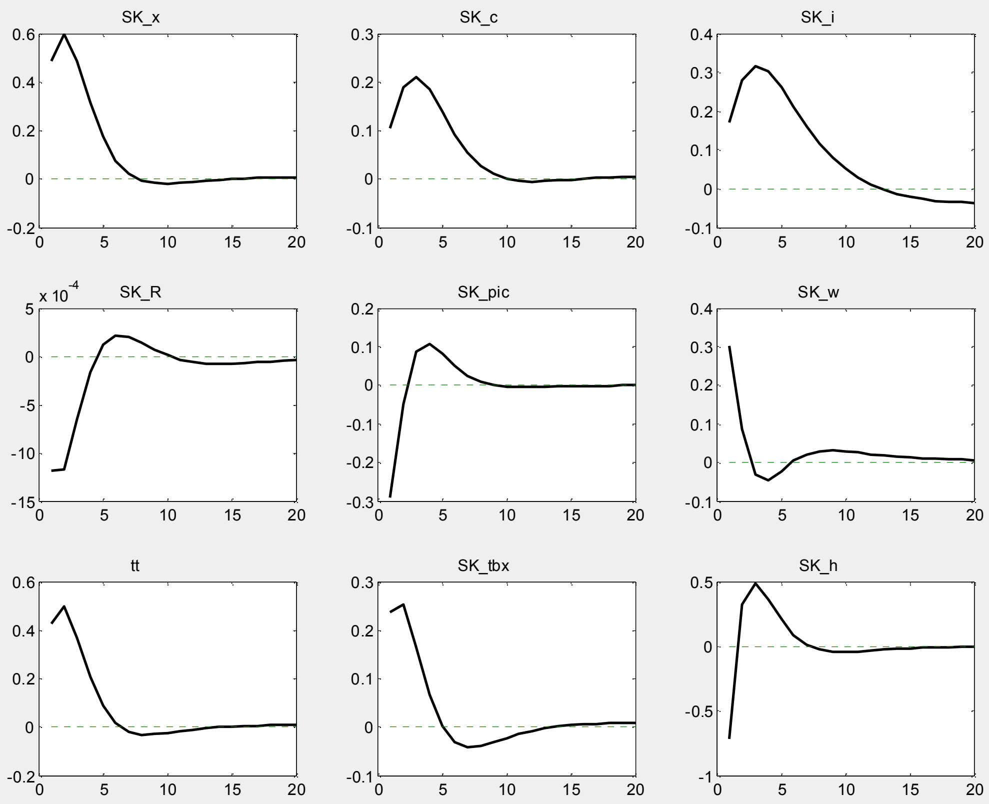
Note: Drawn at posterior mean (solid) with 40% (dark shaded) and 80% (light shaded) intervals. Vertical axes correspond to percentage deviations from the steady state. There is an exception for interest rates (R) and inflation (pic) where vertical axes correspond to percentage points deviations.

Overall, these effects result in output rising by about 0.5% shortly after the shock hits the economy. Consumer inflation declines by about 0.25 percentage point. Monetary policy in response to developments in the economy loosens its stance and the nominal interest rate declines by 0.07 percentage point.

Monetary union

The exchange rate channel in the regime of autonomous monetary policy creates an amplifying effect. Just as foreign demand rises more slowly when the exchange rate is not allowed to change, so too is the positive wealth effect weaker. The only exception is the substitution effect. Nominal interest rates are unchanged while the prices fall more, which taken together lead to lower real interest rates and motivate households to spend more.

Figure 5 Technology shock – common monetary policy



4.3 MONETARY POLICY SHOCK

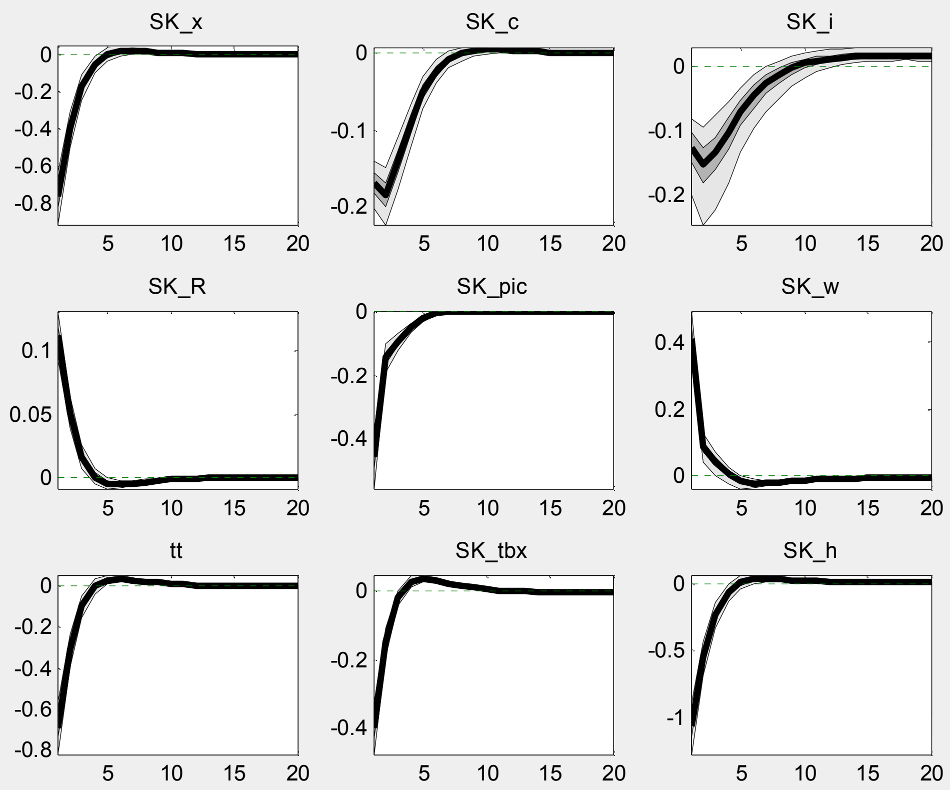
Autonomous monetary policy

The monetary policy shock represents an unexpected 100 basis point (in annualised terms) tightening of the monetary policy stance. Unlike with the previous two simulations there is no inertia in the shock. The path of the interest rate from the second period on is purely an endogenous reaction of the rate to developments in the economy.

In this simulation, it is apparent that the exchange rate plays a crucial role. As the interest rate increases, the exchange rate tends to strongly appreciate, at the same time accelerating and amplifying the reaction of home variables. Recall that we do not have any adjustment costs in the model that mitigate the reaction of the exchange rate to shocks. The responses of the variables immediately reach their maximum impacts and there are not the usual hump-shaped patterns.

The tightening of the monetary policy results in a higher cost of borrowing and in households postponing consumption. In order to meet lower demand firms, reduce labour. The resulting fall in output leads to lower marginal productivity of factors of production, which in turn means lower return on capital. As a consequence households cut investment and thus further decrease output. As noted in the previous paragraph, the exchange rate strongly appreciates and puts downward pressure on import prices. Domestically produced goods are replaced by imports. On the other hand, prices of domestic products on the foreign market become expensive and foreign consumers switch to higher consumption of their domestically produced goods. Consequently, the export performance of the home economy declines and the trade balance worsens considerably. The negative effect on households' consumption is partly offset by a rise in the real wage. This increase is a somewhat surprising result, while the modest movements in the nominal wage (due to its considerable stickiness) are dominated by the rapid decrease in consumer inflation.

Figure 6 Monetary policy shock – autonomous monetary policy



Note: Drawn at posterior mean (solid) with 40% (dark shaded) and 80% (light shaded) intervals. Vertical axes correspond to percentage deviations from the steady state. There is an exception for interest rates (R) and inflation (pic) where vertical axes correspond to percentage points deviations.

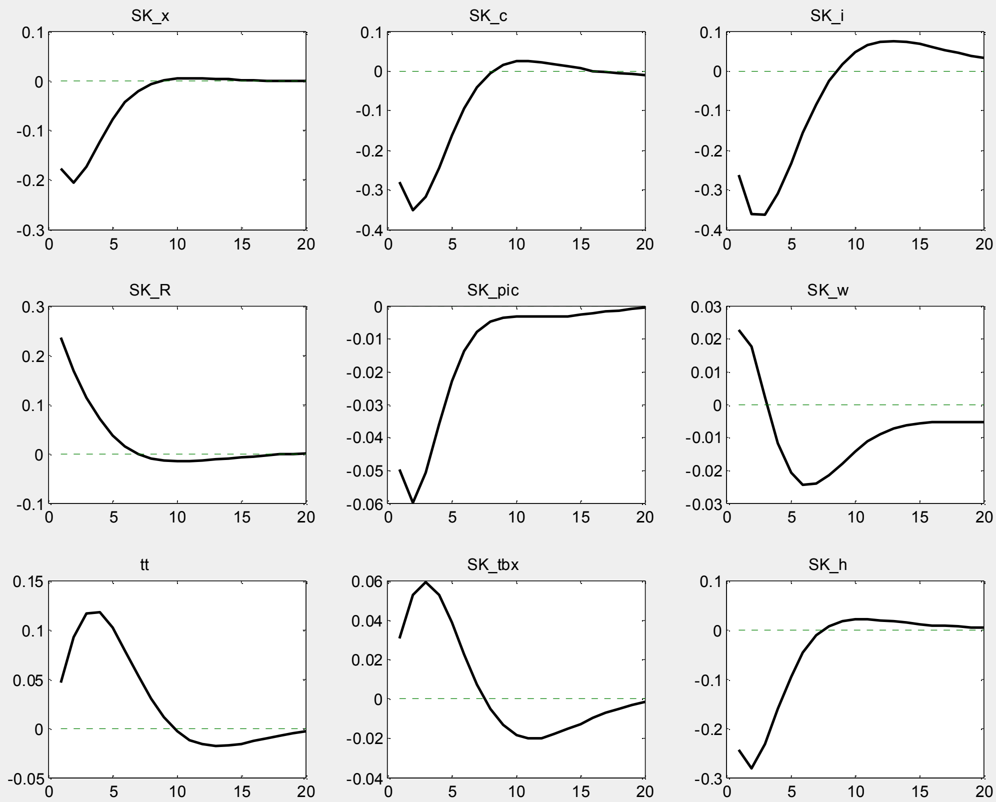
Taken together, the exchange rate strongly amplifies the effects of the monetary policy shock when domestically produced goods are replaced by foreign production. The maximum impact on output is about 0.7%. Consumption good inflation is affected as well, when its decline, 0.4 percentage point, significantly exceeds the fall in the inflation of intermediate good prices.

Monetary union

The monetary policy shock in the regime of monetary union is implemented as an unexpected one-off 100 basis points shock to the common union-wide (annualised) interest rate. An important difference in the setup of this simulation is that, in this case, the shock hits both regions in the model in line with the monetary union definition.

In quantitative terms, the responses of the variables are far different from the responses in the above simulation. The absence of exchange rate movements causes a drop in both exports and imports. The impact is, however, much smaller. Due to the evolution of the terms of trade, the overall effect is such that the trade balance improves initially. The absence of the exchange rate effect is clearly visible in other variables, too. Lower imports lead to a pronounced decrease in consumption and investment, while the effect on consumer prices is much weaker. On the other hand, the slower decline in exports results in a lower fall in total output and consequently in labour. Moreover, as other studies have also found, the real wage now follows the pattern in output.

Figure 7 Monetary policy shock – common monetary policy



The impulse-response functions of investment shock and preference shock to consumption are presented in the Appendix.

5 CONCLUSION

In this paper, we have described a two-country DSGE model suitable for policy analysis of the Slovak economy as a part of the euro area. There are several standard features incorporated in the model, such as external habit formation, investment adjustment costs, sticky prices and wages, and flexible capital utilisation. The model allows switching between two types of monetary regime. In one regime, the model can be specified for two countries that each have an autonomous monetary policy; in the other regime, the two countries constitute the euro area with a common monetary policy.

The possibility of regime switching is a very practical tool especially for countries that have joined a monetary union in recent history, as is the case with Slovakia. Utilising this type of model and quarterly data covering the years 1997 to 2008, we were able to estimate selected parameters of the model. The parameterisation of the model consisted of two steps. Firstly, all parameters controlling the steady state were determined. Here, the aim was to match the deep ratios (such as the ratios of investment, government, and trade to output) determined by the model and computed from the empirical dataset. Secondly, the remaining structural parameters and all parameters describing the structural shocks were estimated via Bayesian method.

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7 APPENDIX

Table 3 Calibrated parameters

Parameter	SK	EAA	Description
Households			
hab	0.64	0.70	degree of habit persistence
β	0.998	0.998	subjective discount factor
δ	0.02	0.02	depreciation rate of capital
v	2	2	inverse of the Frisch elasticity of labour supply
n	0.01	0.99	country size
Intermediate-good firms			
α	0.7	0.7	share of labour income in production
g^A	1.004	1.004	steady state growth
Distributors			
ω_C	0.638	0.997	home bias in production of final consumption goods
ω_I	0.441	0.998	home bias in production of final investment goods
μ_I	2	2	price elasticity of demand for investment goods
μ_C	2	2	price elasticity of demand for consumption goods
Adjustment costs			
$\gamma_{u,2}$	0.26	0.02	parameter of capital utilisation cost function
σ_d	6.0	3.7	elasticity of substitution between differentiated intermediate goods
σ_w	6	6	elasticity of substitution between labour services

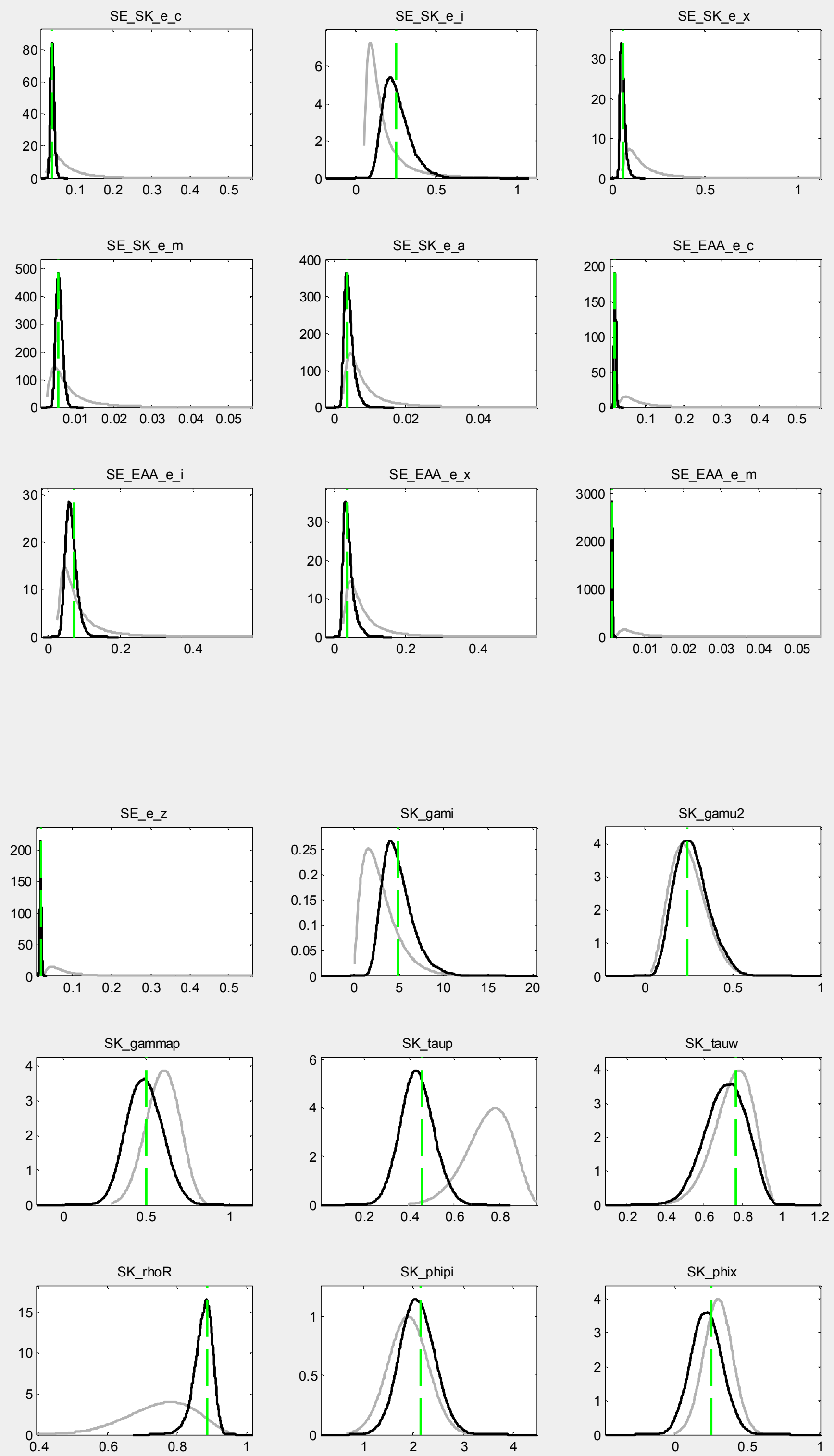
Table 4 Prior distribution

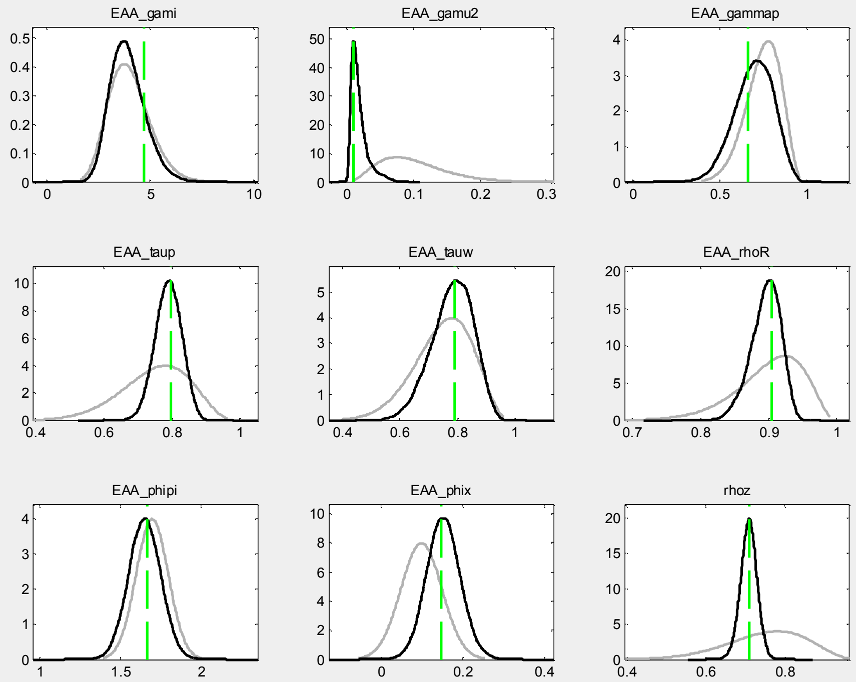
Parameter		Distribution	Mean	St. dev.
Slovakia				
Adjustment costs				
Parameter of investment adj. cost	γ_I	Gamma	3.00	2.00
Parameter of capital utilisation cost	$\gamma_{u,2}$	Beta	0.25	0.10
Wage and price setting				
Price indexation	γ_p	Beta	0.60	0.10
Probability of no price change	τ_d	Beta	0.75	0.10
Probability of no wage change	τ_w	Beta	0.75	0.10
Taylor rule				
Interest rate smoothing	ρ_R	Beta	0.75	0.10
Resp. to inflation	ϕ_π	Normal	1.90	0.40
Resp. to output growth	ϕ_x	Normal	0.30	0.10
Autoregressive coefficients				
Preference shock	ρ_C	Beta	0.50	0.20
Investment shock	ρ_I	Beta	0.50	0.20
Covariance stationary technology shock	ρ_X	Beta	0.85	0.10
Technology growth shock	ρ_A	Beta	0.50	0.20
Asymmetric technology innovation	ρ_Z	Beta	0.75	0.10
Standard deviations				
Preference shock		Inverse-gamma	0.10	Inf.
Investment shock		Inverse-gamma	0.20	Inf.
Cov. stat. techn. shock		Inverse-gamma	0.20	Inf.
Monetary shock		Inverse-gamma	0.01	Inf.
Technology growth shock		Inverse-gamma	0.01	Inf.
Asymmetric technology innovation		Inverse-gamma	0.10	Inf.
Euro area				
Adjustment costs				
Parameter of investment adj. cost	γ_I^*	Gamma	4.00	1.00
Parameter of capital utilisation cost	$\gamma_{u,2}^*$	Beta	0.1	0.05
Wage and price setting				
Price indexation	γ_p^*	Beta	0.75	0.10

Probability of no price change	τ_d^*	Beta	0.75	0.10
probability of no wage change	τ_w^*	Beta	0.75	0.10
Taylor rule				
Interest rate smoothing	ρ_R^*	Beta	0.90	0.05
Resp. to inflation	ϕ_π^*	Normal	1.70	0.10
Resp. to output growth	ϕ_x^*	Normal	0.10	0.05
Autoregressive coefficients				
Preference shock	ρ_C^*	Beta	0.50	0.20
Investment shock	ρ_I^*	Beta	0.75	0.10
Covariance stationary technology shock	ρ_X^*	Beta	0.85	0.10
Standard deviations				
Preference shock		Inverse-gamma	0.10	Inf.
Investment shock		Inverse-gamma	0.10	Inf.
Cov. stat. techn. shock		Inverse-gamma	0.10	Inf.
Monetary shock		Inverse-gamma	0.01	Inf.

Note: Confidence interval equals mean ±2 st. dev.*

Figure 8 Prior and posterior distribution




EAA_gami
EAA_gamu2
EAA_gammap
EAA_taup
EAA_tauw
EAA_rhoR
EAA_phipi
EAA_phix
rhoz

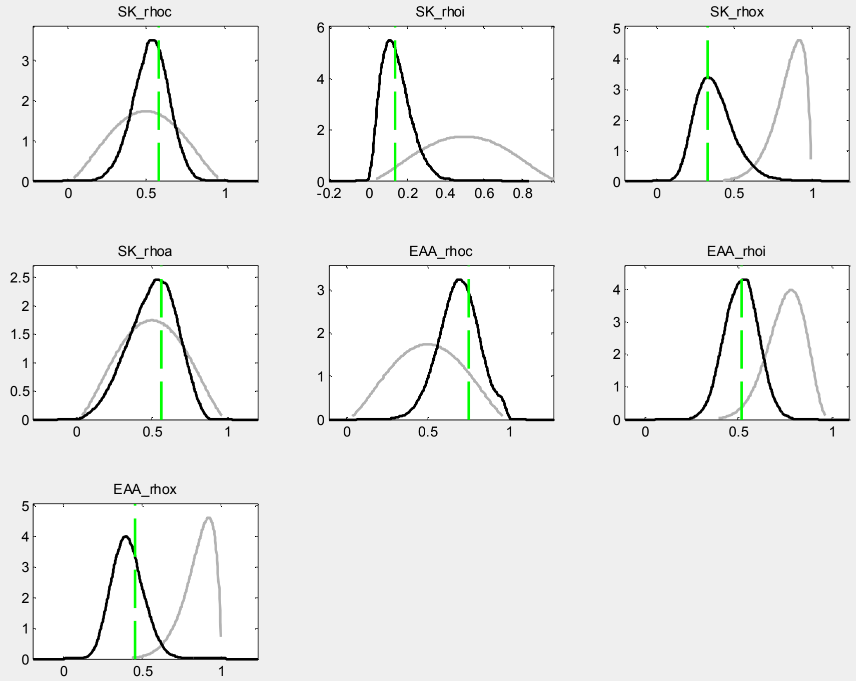
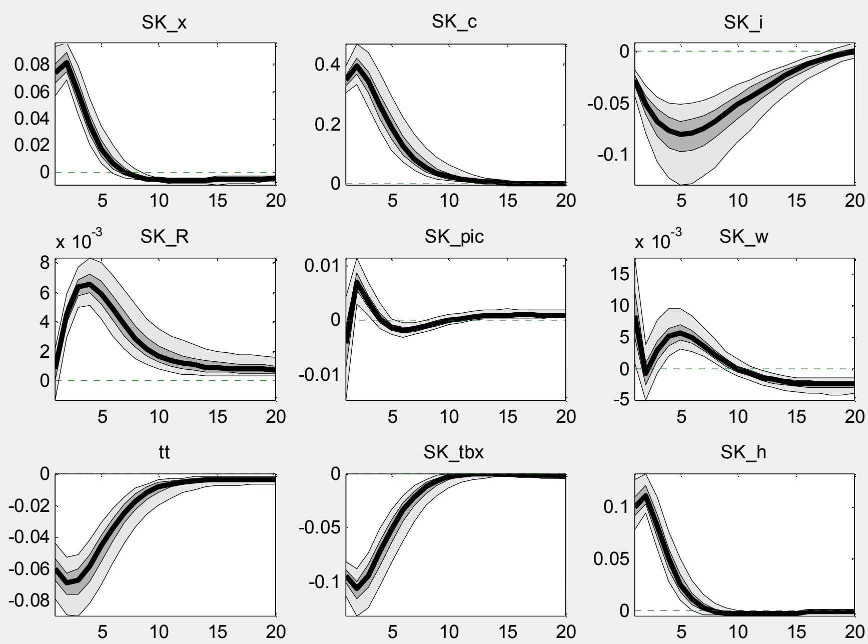

SK_rhoc
SK_rhoi
SK_rhox
SK_rhoa
EAA_rhoc
EAA_rhoi
EAA_rhox

Figure 9 Consumption preference shock– autonomous monetary policy



Note: Drawn at posterior mean (solid) with 40% (dark shaded) and 80% (light shaded) intervals. Vertical axes correspond to percentage deviations from the steady state. There is an exception for interest rates (R) and inflation (pic) where vertical axes correspond to percentage points deviations.

Figure 10 Consumption preference shock – common monetary policy

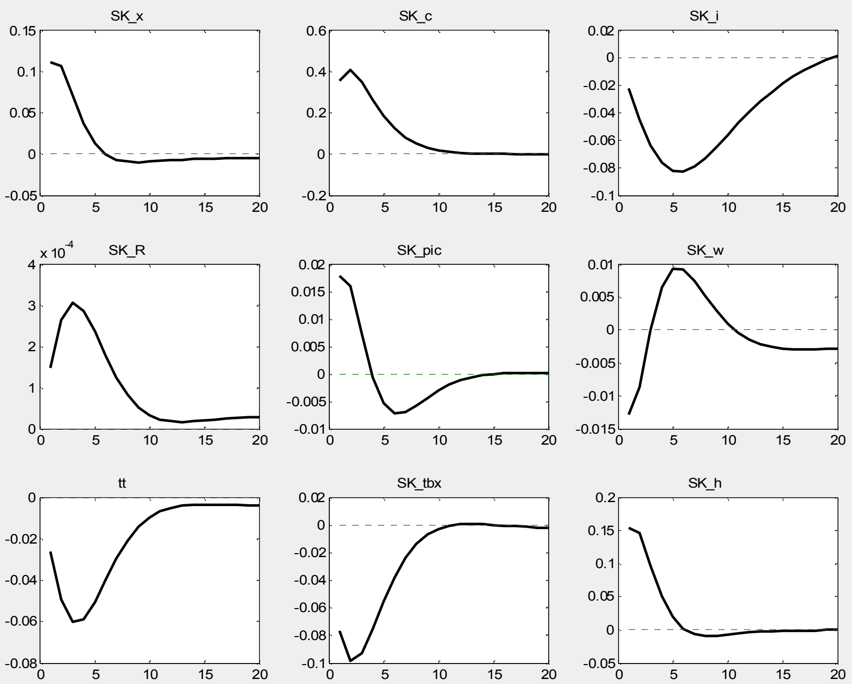
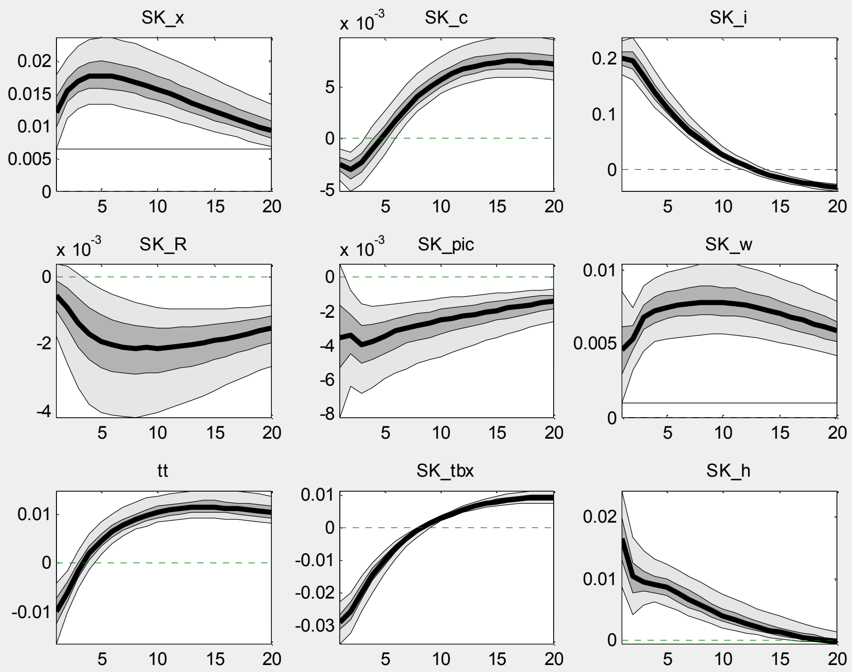


Figure 11 Investment shock – autonomous monetary policy



Note: Drawn at posterior mean (solid) with 40% (dark shaded) and 80% (light shaded) intervals. Vertical axes correspond to percentage deviations from the steady state. There is an exception for interest rates (R) and inflation (pic) where vertical axes correspond to percentage points deviations.

Figure 12 Investment shock – common monetary policy