TIME INCONSISTENCY IN MONETARY POLICY

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Since the times of accepting the state in the economic environment, which is linked mainly with the “New Deal” programme, a polemic has been raging over the magnitude and justification of such interventions in an economy. However, a stabilisation policy, which should ensure permanently sustainable growth, is sometimes subordinated by the state to short-term aims, which later require painful re-adjustments. If we work from the assumption that permanent growth of the nominal money supply is the main source of inflation, for understanding the causes of high inflation it is necessary to know the causes of a high growth in the money supply. For advanced economies, where the issuance of money does not play a significant role in government revenues, the main reason is the existence of a compromise between unemployment and inflation. If a policymaker believes that he is able to influence real income by movements in aggregate demand, then an expansionary monetary policy may reduce unemployment.

Such a policy is clearly short-termist. From the long-term aspect however this “trade-off” between inflation and unemployment does not apply. This implies that average inflation does not have an influence on average unemployment and the average real income. If we compare two policies, where one differs from the other only in terms of the constant rate of growth in the money supply, then under the assumption that all agents are aware of these conditions in the economy, there is no reason for real income to behave differently in an economy with low inflation than in an economy with high inflation. The revolutionary work of Kydland and Prescott (1977) however highlighted the fact that even in the case of the non-existence of a long-term relationship between inflation and unemployment an unclear and misleading policy can lead to high inflation without positive effects on unemployment.

The aim of this work is to clarify the reasons and consequences of such behaviour by a policymaker in the economy. In the introduction the realisation of monetary policy is described as a non-cooperative game between the policymaker and the public, where the public perceives the commitments of politicians and on the basis of this information forms expectations. Later this model is enriched by rational expectations. Later this model is enriched by rational expectations. The reason is clear. The government through its decisions influences the public’s expectations. The government, however, always has the temptation to mislead the public, so that the public formulates lower inflationary expectations. The reason is clear. The government finances its deficits largely by issuing bonds. The nominal interest rate \( i \), which the public accepts in purchasing government bonds, is determined by the required rate of expected real interest rate \( r_e \) and the expected change in the price level \( \pi^e \):

\[
i = r_e + \pi^e \quad (1)
\]

The Government can sell bonds at a low nominal rate, only if the public expects low inflation. In the case of the real rate \( r \), paid by the government, the real costs to the government are given by the relationship:

\[
r = i - \pi, \quad (2)
\]

where \( \pi \) represents actual inflation. If it is in the hands of the politician to determine a change in the price level \( \pi \), so that he achieves lower real costs, he needs to increase inflation above the public’s expectations. From the equations 1 and 2 we reach:

\[
r = r_e + \pi^e - \pi \quad (3)
\]

From the equation above it is clear why the government has the temptation to increase inflation above the public’s expectations. If a politician sets \( \pi^e > \pi \), then \( r < r_e \), i.e. by abandoning the commitment to adhere to the promised rate of inflation he gains in the case of higher inflation revenue from the lower real interest rate, which the public did not
expect. This forces the politician to assert a strategy where inflationary expectations are lower than the actual rate of inflation.

**Rules or discretionary decisions**

The standard rule of monetary policy says that at any moment it is necessary to choose the best policy that can be selected. Kydland and Prescott (1977) elaborated this theory, where they sought such a policy that would be consistent from the time aspect. According to them the monetary rule is a certain form of promise, which the policymaker endeavours to adhere to in order that the public’s expectations are in line with the actual rate of inflation. Conversely, discrete decisions desist from monetary-policy rules and monetary policy comes to follow its own aims. Let us assume the linear relationship between unemployment and inflation:

\[ u_t = \alpha (\pi^e_t - \pi_t) + u^*_t \]  

(4)

where \( u_t \) is the unemployment at time \( t \), \( \pi^e_t \) the natural rate of unemployment, \( \pi_t \) actual inflation and \( \alpha \) a parameter. It is assumed that expectations are rational, \( \pi^e_t = E \pi_t \), and the optimal rate of inflation equals 0. The aim of the policy is to minimize the certain loss function, in our case defined as the deviation from the natural rate of unemployment and the rate of inflation:

\[ S = f(\pi_t, u_t - u^*_t) \]  

(5)

We can minimize the loss function subject to (4). In this case any deviation from the optimal values \( \pi_t = 0 \) and \( u_t = u^*_t \) represents an undesirable loss for the policymaker.

**Figure 1**

![Phillips' curve](image)

From Figure 1 it is clear that a minimisation of the loss function for the policymaker is a search for the optimal rate of inflation. Let us assume that at the beginning the economy is found in “an optimal state” \( E_0 \) (zero inflation, and the natural rate of unemployment). This point however does not correspond to the condition of optimality in the given Phillips curve. The public is indifferent as to inflation and unemployment (a movement along the Phillips curve) and therefore it is advantageous for the policymaker to reduce unemployment (the state \( E_1 \) through an unexpected increase in inflation (a discrete decision). If however agents, i.e. market subjects, perceive higher inflation, they adapt their expectations to this and the economy remains at the point \( E_2 \). This point however already corresponds to the indifference curve with higher losses. The politician in the end with the aim of reducing the loss achieves the opposite effect, since the targeted increase in inflation did not in the end bring any positive effect in unemployment. The effectiveness of the policy in this case is determined by rational or adaptive expectations. If agents were to form adaptive expectations, then a transition from the point \( E_0 \) to \( E_1 \) corresponds to an expansionary policy. If however the agents understand the politician’s effort to influence economy, the result will be point \( E_2 \) that will be worse than \( E_0 \) and \( E_1 \). In this case a discrete policy appears to be worse than the policy of the rule of adhering to stable inflation. Barro and Gordon (1983) elaborated this theory further, with the aim of highlighting the discordance between the policy of the rule in models with the rational expectations and experience of market economies, where pragmatic politics prevails. Their model is based on the assumption that a certain optimal level of inflation \( \pi^* \) exists in an economy. In the case that the economy deviates from the given inflation, they define this deviation as a certain cost for the economy.

\[ \text{cost} = \alpha(\pi - \pi^*)^2 \]  

(6)

Figure 2

![Cost function](image)

A clarification of the deviation of optimal values of \( \pi^* \) is based on the knowledge that in the case of higher inflation the economy suffers too high transaction costs and in the case of lower inflation the seignorage is very low. As however was indicated in the example with speculation in the sale of bonds, a politician in selecting a higher-than-expected inflation has a certain advantage, which ensues from lower real interest rates or lower real debt.

Let us assume that we can value this advantage and the concave revenue function corresponds to it:

\[ \text{revenue} = b(\pi - \pi^*) - c(\pi - \pi^*)^2 \]  

(7)
The course of the revenue function is graphically depicted in figure 3.

**Figure 3**

![Graph of Total Revenue](image)

The politician, naturally, wants to set a rate of inflation that would minimise the total losses \( L \), i.e. the difference between the costs and revenues:

\[
L = a(\pi - \pi^*)^2 - [b(\pi - \pi^*) - c(\pi - \pi^*)^2]
\]

where \( a, b, c > 0 \). The politician thus decides on the level of inflation \( \pi \) on the basis of information on the public’s expectations \( \pi_e \). The public’s expectations in this case influence the politician’s decision, who can however through an appropriate strategy anchor expectations by a certain promise. Through differentiating the equation (8) we get the first order condition for optimal choice:

\[
\frac{dL}{d\pi} = 2a(\pi - \pi^*) - b + 2c(\pi - \pi^*) = 0,
\]

from which:

\[
\pi = \frac{-b/2 + c\pi^*}{a + c},
\]

which in this case represents the policymaker’s reaction function.

From this equation it is apparent that with a growth in inflationary expectations, the resultant inflation also grows and will be higher than its optimal level \( \pi^* \). The coefficient \( b \) represents the marginal revenue in the case of growing inflation (above the level of inflationary expectations) and is characteristic prevailingly for countries with a high level of debt. From the long-term aspect however a view on the correlation between inflationary expectations and actual inflation is important. Are expectations actually rational and if yes, what value is beneficial for the economy?

Let us compare several scenarios of inflationary expectations and the loss in the case of these scenarios.

**Scenario 1: Inflationary expectations equal zero**

By substituting \( \pi_e = 0 \) into the reaction function we get:

\[
\pi^* = 0 \Rightarrow \pi = \frac{a\pi^* + b/2}{a + c} > 0
\]

These inflationary expectations are not rational. The public in its decision-making on inflation does not have any relevant information that inflation will be zero, and therefore this solution in the conditions of rational decision-making is inadmissible.

**Scenario 2: A discrete decision with the promise of optimal inflation**

This situation can occur, if the politician is able to influence the public’s expectations by a certain promise that inflation will equal its optimal value, i.e. \( \pi = \pi^* \). The public thereby forms its expectations on the basis of the politician’s promise \( \pi^* = \pi^* \). If the politician knows that the public expects the optimal level of inflation \( \pi^* \), he can make a discrete decision with the aim of minimising the overall loss.

\[
\pi^* = \pi^* \Rightarrow \pi = \frac{a\pi^* + b/2 + c\pi^*}{a + c} = \pi^*, \quad \pi^* \leq \pi_N^{NAIVE} > \pi^* \]

Then the overall loss defined by the relationship (8) after substituting \( \pi \) and \( \pi^* \) is:

\[
L_N^{NAIVE} = -\frac{b^2}{4(a + c)} < 0
\]

The resultant inflation is higher than its optimal level. The public set its expectations with the knowledge that the policymaker would keep his promise. However, he reneged on his promise and thus agents lost confidence that the politician’s future promise would be kept. The politician, through his decision, increased inflation above the optimal level, whereby he achieved a reduction in the real debt without the need to raise taxes. Rational agents however realised that their expectations had not been correct.

**Scenario 3: A discrete decision and rational behaviour of the public**

The public realises the politician’s efforts to deflect its expectations from actual inflation and thereby the promise
loses any import and the agents carefully monitor the politician's plans and formulate expectations according to actual inflation $\pi^e = \pi$. The point in Figure 4 corresponds to these expectations, in which the reaction function cross the 45° line. Through substituting into the reaction equation and following an adjustment we get:

$$\pi^e = \pi^* \Rightarrow \pi = \frac{a\pi^* + b}{a} = \pi^* + \frac{b}{2a} = \pi^{RE} > \pi^* \quad (15)$$

and following substitution into the function of the loss:

$$L^{RE} = \frac{b^2}{4a} > 0 \quad (16)$$

The resultant inflation is higher than its optimal level for two reasons:
1. The politician still sees the possibility of revenue from inflation higher than expectations. This forces him to undertake an expansive policy.
2. The public however understand such speculative thinking by the politician. If in the case of the expectations $\pi^e = \pi^*$ actual inflation was higher, then the public will also form its expectations at a level higher than the optimal rate of inflation.

Through comparing these two scenarios we come to the interesting conclusions:

$$\pi^{RE} > \pi^{NAIVE} > \pi^e \quad L^{RE} > L^{NAIVE} \quad (17)$$

The rational behaviour of agents in this case is worse than if they "naively" believed the policymaker’s promise. In both cases inflation is deflected from its optimal value. In the case of scenario 3 inflation is higher than its optimal level, though in comparison with scenario 2 the politician in his discrete behaviour does not have any revenue from the additional inflation, which is already consistent with expectations.

There is, however, a certain solution that can eliminate the losses ensuing from the rational behaviour of agents. Barro and Gordon expressed the opinion that it is necessary for policymaker’s “tied the hands”. Instead of a discrete choice of inflation, the politician should commit himself to complying with certain rules in seeking an optimal rate of inflation and should not have the temptation to renege on this promise.

**Scenario 4: A credible rule**

The let us assume that the politician passes a law and has expressed the promise that he will always set a rate of inflation equalling its optimal value $\pi = \pi^*$. In this scenario he will no longer be tempted to renege on his promise. Agents in this case will believe the promise and also set $\pi^e = \pi^*$. We can then calculate the loss:

$$L^{RULE} = 0 \quad (18)$$

Through comparing the results of the individual scenarios we reach:

$$L^{RE} > L^{RULE} > L^{NAIVE} \quad (19)$$

A scenario 2 represents the least losses, where the net loss is negative. This solution however is possible only in adaptive expectations or in the case that the politician’s promise is sufficiently reliable for agents to form their expectations on the basis of it. With each politician’s promise broken his credibility declines and agents form their expectations according to scenario 3. Here the loss is already worse, and therefore the better solution appears to be a rule-based policy. The public in this case believes the promise and the politician, in the interest of complying with the rule, does not change the rate of inflation, to which he is bound to adhere. Nevertheless, the problem lies in the fact that a politician with sufficient credibility, which has been achieved by credible rule, still has the temptation to mislead the public in order to gain some benefit through an inflationary shock. Such a solution with the help of the rule-based policy is thus optimal, but time inconsistent. From the short-term point of view the politician can achieve a reduction in unemployment, but this is later manifested in a growth in inflation. By contrast, the discrete solution with rational agents is time consistent, but sub-optimal. If we wanted to achieve credible rule, then it would be necessary to find such a rate of inflation where the politician would not be tempted to break this rule.

**Applying the Rule-Based Policy**

So far we have dealt with the behaviour of an economy in a static manner. If we work from the assumption that at the time $t$ agents form their expectations on the basis of a politician’s credibility, about whom they know that in past he has behaved with the help of the rule, then their expectations concerning inflation at the time $t$ are $\pi_t^e = \pi_t^*$. The politician in this case has two strategies:

1. To comply with the rule also at the time $t$ and set inflation $\pi_t = \pi_t^*$.
2. To breach the rule and make a discrete decision according to (13).

This situation is presented in the following table:

<table>
<thead>
<tr>
<th></th>
<th>$t$</th>
<th>$t + 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Public’s expectations $\pi_t^e$</td>
<td>$\pi_t^*$</td>
<td>$\pi_{t+1}^*$</td>
</tr>
<tr>
<td>Actual inflation $\pi_t$</td>
<td>$\pi_t = \pi_t^*$</td>
<td>$\pi_{t+1} = \pi_{t+1}^*$</td>
</tr>
<tr>
<td>Loss $L_1$</td>
<td>$L_{1,t} = 0$</td>
<td>$L_{1,t+1} = 0$</td>
</tr>
<tr>
<td>2. Public’s expectations $\pi_t^e$</td>
<td>$\pi_t^*$</td>
<td>$\pi_{t+1}^*$</td>
</tr>
<tr>
<td>Actual inflation $\pi_t^*$</td>
<td>$\pi_t = \pi_t^* + \frac{b}{2(a + c)}$</td>
<td>$\pi_{t+1} = \pi_{t+1}^* + \frac{b}{2a}$</td>
</tr>
<tr>
<td>Loss $L_2$</td>
<td>$L_{2,t} = -\frac{b^2}{4(a + c)}$</td>
<td>$L_{2,t+1} = \frac{b^2}{4a}$</td>
</tr>
</tbody>
</table>
If the politician breaches the rule at the time \( t \), he can expect that agents will stop to believe the rule-based policy (loss of credibility) and in future periods they will form their expectations not according to the promise, but rationally.

The politician discounts the expected loss at the time \( t + 1 \) by the factor:

\[
q = \frac{1}{1 + r},
\]

where \( r \) is the discount rate. The decision as to whether to execute the first or the second strategy is given by the expected total loss \( L_T \).

\[
L_T = L_t + qL_{t+1},
\]

Let us now define the variable temptation \( T \) as the difference between the losses in the case of strategies 1 and 2 at time \( t \).

\[
T = L_t^{RULE} - L_t^{NAIVE} = 0 - \left( -\frac{b^2}{4(a + c)} \right) = \frac{b^2}{4(a + c)} > 0 \quad (22)
\]

The variable \( T \) takes positive values; the politician is tempted to execute strategy 2. The temptation thus quantifies the expected loss (or gain) in the case of a change of strategy. Through an inflationary shock however at the time \( t + 1 \) a change in the public’s expectations would occur and thereby also a worse result at the time \( t + 1 \) would result.

Let us define the term enforcement \( P \), which rates the advantageousness of the rule over a discrete decision made at the time \( t + 1 \). Mathematically this means the discounted difference between the losses in the case of strategy 2 over strategy 1 at the time \( t + 1 \).

\[
P = q(L_{t+1}^{RULE} - L_{t+1}^{NAIVE}) = q \left( \frac{b^2}{4a} - 0 \right) = \frac{q b^2}{4a} > 0 \quad (23)
\]

The variable \( P \) also takes positive values, so in this case the loss even after discounting at the time \( t + 1 \) is greater in a discrete policy than in adhering to the rule.

Thus we reach the search for a suitable strategy for the politician. If the rule is to be more successful than a discrete decision, the rating of the preference for the rule \( P \) must be greater than the preference for the potential benefit in the case of the discrete decision \( T \). In other words if the loss from abandoning the rule is greater than the benefit ensuing from applying a discrete policy, there is no reason to break the rule.

\[
T \leq P \quad (24)
\]

after substituting

\[
\frac{b^2}{4(a + c)} \leq \frac{q b^2}{4a} \quad (25)
\]

from which for the value \( q \) we get:

\[
q \geq \frac{a}{a + c} \quad (26)
\]

The discount rate \( r \), which determines \( q \) reflects the policymaker’s ideas of the future. If he takes a short-termist view in his considerations\(^1\), then he chooses a higher \( r \), whereby \( q \) approaches zero.

On the other hand:

\[
a \gg c \Rightarrow \lim_{[a - c] \to +} \frac{a}{a + c} = 1 \quad (27)
\]

There is a contention here between the preference weightings \( a \) and \( c \) and the discount factor and the discount factor \( q \). Whereby the weighting of \( a \) and \( c \) draw apart, then in the short term the rule-based policy of adhering to \( \pi^* \) is ever more threatened, allowing room for temptation to execute a discrete decision. And this threatens adherence to the rule-based policy.

If we accept the assumption that \( c = 0 \), enforcement of the rule is impossible. From this it results that the rule of adhering to the optimal rate of inflation \( \pi^* \) equilibrium will not be will not be an equilibrium solution to this model.

**Best rule and central bank equilibrium**

As the foregoing finding showed, that in the case of the credible rule the optimal rate of inflation \( \pi^* \) no equilibrium will be ensured, we can generalise our analysis for the rule of adhering to the set rate of inflation \( \pi^N \), which need not correspond to the optimal rate of inflation. Thereby agents’ expectations are also changed, which in the case of a credible rule will be \( \pi^r = \pi^N \).

For simplification let us also assume \( c = 0.\)\(^2\) Then let us reformulate the loss functions (14) and (18).

\[
L_t^{NAIVE} = \frac{b^2}{4a} - b \left[ (\pi^* - \pi^N) + \frac{b}{2a} \right] \quad (28)
\]

\[
L_t^{RULE} = \alpha (\pi^N - \pi)^2 \quad (29)
\]

\[
L_t^{RE} = \frac{b}{4a} \quad (30)
\]

Through a modification of the equations (14) and (18) we get also a generalisation for the equations (21) and (22).

\[
T = \frac{b}{2a} \left[ (\pi^N - \pi)^2 \right] \quad (31)
\]

\[
P = q \left[ \frac{b}{2a} \right]^2 - (\pi^N - \pi)^2 \quad (32)
\]

The course of these two curves is depicted in Figure 5 and it can be seen from it that if the politician selects the

\(^1\) For example due to approaching elections or high risks in the future.

\(^2\) This assumption does not influence the conclusions ensuing from the analysis, it can merely change the resultant analytical solution.
rule in the case of inflation $\pi^\ast$, the temptation from applying a discrete decision is greater than adhering to the rule-based policy. With a growth in inflation, the benefit from an inflationary shock declines, and thereby the politician’s temptation to execute a discrete policy also declines. The minimum is reached at the point $D$, which represents inflation in the case of a discrete rule. At this point the rule-based policy and discrete policy are realised with the same inflation, whereby the temptation is zero. In the case of a rule-based policy and discrete policy are realised with the same inflation, thereby the temptation again grows due to the lower losses in applying discrete decisions than in the case of a rule-based policy and the politician prefers a discrete rule with the inflation $\pi^{RE}$.

**Figure 5**

From the course of both curves it can be seen for what levels of inflation $\pi$ is the rule-based policy better than a discrete decision. In this case it is the interval:

$$\pi^E < \pi < \pi^{RE}$$  \hspace{1cm} (33)

At this level of inflation the temptation to apply a discrete policy is already less than the enforcement of adherence to a rule-based policy. Through the solution of the system $T = P$ we get the critical values for inflation $\pi$, for which the policy of adhering to inflation set in advance is an equilibrium strategy in comparison with the application of discrete decisions and re-seeking lost credibility.

$$\pi^E = \pi^\ast + \frac{1 - q}{1 + q} \frac{b}{2a}$$  \hspace{1cm} (34)

$$\pi^{RE} = \pi^\ast + \frac{b}{2a}$$  \hspace{1cm} (35)

Within this interval we can find a rate of inflation for which minimize the loss. In our understanding this is represented by point $E$, where the politician realises the equilibrium inflation $\pi^E$ and the loss:

$$\pi^E = \pi^\ast + \frac{1 - q}{1 + q} \frac{b}{2a} \Rightarrow L(\pi^E) = \frac{b^2}{4a} \frac{(1 - q)^2}{1 + q}$$  \hspace{1cm} (36)

This condition of equilibrium determines for us the choice of the rate of inflation at which agents form the same inflationary expectations and the politician does not have a tendency to abandon the rule. If he were to do so, he would lose credibility and agents would no longer believe his promise. They would form higher inflationary expectations and thereby additional costs arise connected with higher inflation, which are, even after discounting, higher than the initial benefits from the inflationary shock. The politician therefore always endeavours to balance out the benefits from unexpected inflation in the short term and the costs connected with the loss of credibility.

We have thus found a stable solution for applying a rule-based policy. The equilibrium rate of inflation is determined by the parameters $a, b$ and the discount factor $q$. If we know that the discount factor is from the interval $0 < q < 1$, the equilibrium rate of inflation depends on the politician’s inclination to give preference to current gains over future losses. So even the equilibrium rate of inflation will be a certain weighted average between the optimal inflation $\pi^\ast$ and the discrete solution $\pi^{RE}$. If $q$ is close to 0, so the politician will give preference more to the present than the future and the current value of future costs will be low, the equilibrium level of inflation will be close to the discrete solution. With the growth of $q \rightarrow 1$ the present value of future losses also grows and the equilibrium rate of inflation approaches the optimal rate of inflation $\pi^\ast$.

Such considerations are however admissible only when we think in terms of an infinite time horizon of a non-cooperative game; otherwise it is, in the recent period, not possible to adhere to a rule-based policy$^3$ and equilibrium is inadmissible. In practice, besides these considerations, there are also added the following requirements for applying a rule-based policy:

- the need to ensure sufficient independence of central banks,
- the requirement that the central bank explicitly subscribes to a certain aim,
- conservatism of the central bank.

**Conclusion**

The aim of this work has been to describe the basic principles of decision-making in applying a rule-based policy or discrete decision-making from the aspect of a non-cooperative game between the public and the politician. The public formulates its inflationary expectations, and the politician on the basis of the expectations determines the growth in

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$^3$This is apparent in the case, where the politician realises that his re-election in the following period is impossible.
the price level. The conflict between short-term and long-term decision-making is in this case determined by the different valuation of future losses in the form of higher inflation. The politician realises that if he wants to maintain an optimal level of inflation by a rule-based policy, he loses a certain benefit from an inflationary shock, which he could execute. Through setting a higher level of inflation that no longer tempts the politician to abandon a rule-based policy we can in this case limit the politician’s temptation. The alternative comprises various measures which prevent the politician from exploiting the benefit from inflation. Only in this way will it be possible for inflation to move in a desirable band, and without unexpected shocks.

References:
9. Sommer, M: Why is inflation not falling in the Czech Republic? Faculty of Social Sciences, Charles University, 1995