NÁRODNÁ BANKA SLOVENSKA

ESTIMATING THE PROBABILITY DISTRIBUTION OF AN INFLATION FORECAST

THE FAN CHART – A GRAPHICAL PRESENTATION

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Forecasting the main macroeconomic indicators is the first step to providing information on where the economy is heading. Each forecast of any economic variable is nevertheless subject to uncertainty, i.e. it bears with it certain risks. These result both from the uncertainty in the development of exogenous variables, as well as from errors ensuing from the simplified model used in preparing the projection. Therefore, potential risks (deviations) are often published together with a forecast,

Graph 1 HICP inflation forecast

where this highlights the alternative deviations to the prediction. The size of these risks may be seen also as the forecast's degree of reliability.
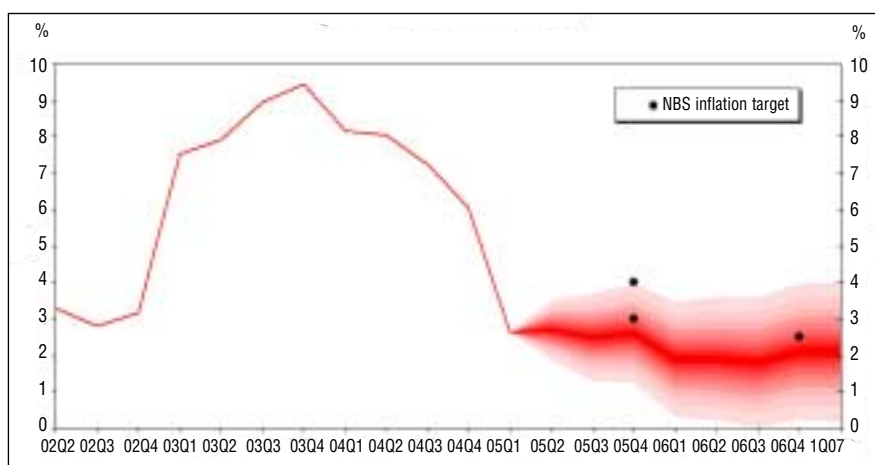
In this article we will attempt to describe the simple approach to estimating the risk inherent in a forecast and to describe the manner of presenting results, where the focus shall be on inflation. One of the ways for presenting a medium-term forecast for inflation together with its possible risks is a graph depicting the estimate of the probability distribution, i.e. the fan chart. Fan charts were first used by the Bank of England in 1996 and thanks to their graphical appeal and easily understandable form, many central banks subsequently started to use them. For the purpose of providing a better understanding of this graph we shall now describe the fan chart depicting the inflation forecast for the SR through to 1Q 2007, taken from the medium-term NBS forecast (April 2005).

The fan chart

The fan chart is a graph depicting estimates of the intervals in which the forecast economic variable (e.g. inflation) will be found. The graph represents an extension of the point forecast by an assessment of the forecast error, this being represented by the width of the interval.

The basis of the graph is the central view (or "base line"), which is the most probable course of inflation for the

given period. This forecast is represented by the darkest line in the middle of the graph. The gradually spreading fan depicts the growth in risks in the central view, highlighting the fact that the degree of uncertainty (forecast error) grows over time. Two equally coloured bands, below and above the central projection represent the extension, of the interval in which the future inflation value will be found, by a size corresponding to the increase in probability by 10% on the preceding interval – confidential intervals[1]. The outermost two bands represent the increase in reliability to the final, 90% level. This means that according to the forecast in Graph 1, inflation at the end of 2006 will, with 90% probability lie within the range 0.2% to 4.0% [2].

Density forecasts

Fan charts, as we have said, depict the estimate of the confidential interval, the width of which represents the size of the risk estimate in the forecast. The confidential interval is derived from the density function (probability distribution). For this reason we often speak of a density forecast. In estimating it we work from the assumption that the density function has known functional form – the func-

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[1] This is not a 10% growth in the interval, but a growth in the interval corresponding to a 10% increase in probability.
[2] The interval 0.2 through 0.4 (the cross-section of the fan) in 4Q of 2006 is a confidential interval.

tion of which depicts the spread of probability – and thereby the prediction is restricted to estimating the parameters of this function. In devising the fan chart we work from the assumption that errors in the forecast may have a skewed normal distribution with the parameters μ, σ a γ [3].

In other words, we need to predict (estimate) two values, the most likely development, i.e. μ, and the size of error in the forecast, i.e. σ, and possibly estimate a third value, γ, which indicates the skewness of error with regard to the central view.

• Central view – represents the best estimate in the probability sense, i.e. the points having the highest probability, the mode. Drawing up the central view for inflation is the first step in obtaining the information necessary for the central bank to determine monetary instrument settings. The central view of inflation for the medium term represents a point estimate created on the basis of outputs from the model simulation and expert estimates.

• σ – standard deviation – represents an estimate of the volatility of the central projection. This is shown in Graph 1 by the size of the confidential interval, i.e. the width of the fan in the given period. The width of the fan for the given quarter is equal to approximately four times the estimate of the standard deviation for the given quarter. The estimate of the standard deviation is an approximation of the uncertainty of the prediction, which may derive either from uncertainty in the development of exogenous variables, or from error in the analytical instrument used (econometric model).

• γ – the balance of risks – is the degree defining what "part of the error" is found below and what part above the central view. This is termed skewness. In Graph 1 the skewness represents a shift in the confidential interval upwards in relation to the central view, while maintaining the interval's size. Oil prices may be used as an example here. Where there is the risk of oil prices being higher, than those given in the projection when preparing the central prediction, the risk of higher inflation is increased. This is represented by a wider interval above the central prediction and a lower interval below it.

Estimates of forecast uncertainty

Methods for estimating uncertainty in a projection may be divided into two groups according to the way the estimate is made. The first group comprises approaches based on estimating uncertainty from an econometric model. This concerns either implicit error in the model, or errors assessed on the basis of shocks input into the model simulation. These shocks may be deterministic or stochastic.

The implicit error of the model is rarely used, since it can be expressed analytically only for certain simple

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[3] A more detailed description is given in the box "Normal density function".

types of forecasting models. Conversely, the approach of estimating uncertainty based on shocks is used frequently. This approach has the inherent advantage of its independence of the length and availability of historical track record. This advantage is a frequent reason for using this approach. On the other hand, its disadvantage is the absence of error in the incorrect setting of shocks. From the philosophy of the approach we can see that it does not take account of the error which may be caused by the incorrect setting of the model itself.

The methods of the second group are based on the assessment of historical track record, this being termed the empirical method of estimating uncertainty. From the aspect of advantages and disadvantages, these methods are quite opposite to those of the first group. The estimate calculated with the help of these methods bears within it almost all the risks connected with the incorrectness of the selection of the model, its subsequent errors and removes the error in the selection of shocks. On the other hand its disadvantage lies in its absolute dependence on the availability of data, and on the idea itself that the future estimate is based on the past. In the case of a transition over to a new method of estimating the projected parameter the result can be inconsistency and a distortion of the estimate.

The empirical method for estimating the standard deviation for the fan chart projection of inflation in the SR

For illustration we have chosen a forecast uncertainty estimate method from the second group, i.e. based on historical errors of inflation forecasts. The main reason and at the same time advantage of using this method is the relatively extensive body of data available from NBS short-term inflation forecasts for the period 2000 – 2004. On the basis of deviations in actual inflation from the projection we can estimate from this source database the forecast uncertainty for the coming four quarters.

For estimating the forecast uncertainty for the first four quarters the statistic RMSE (Root Mean Square Error) was used:

$$RMSE = \sqrt{\frac{1}{n} \sum_{t=1}^{n} e_t^2},$$

where $e_t = \tilde{y}_t - y_t$ is the deviation of the forecast $\tilde{y}_t$ from the observed value y_t at time t.

For estimating the uncertainty of the projection for the first quarter ahead all 3-month forecasts from the period 2000 through 2004 and their deviations were used. The same approach was subsequently applied for estimating uncertainties for two to four quarters ahead.

Due to the insufficient amount of data for two-year projections, the regressive method (estimate using the loga-
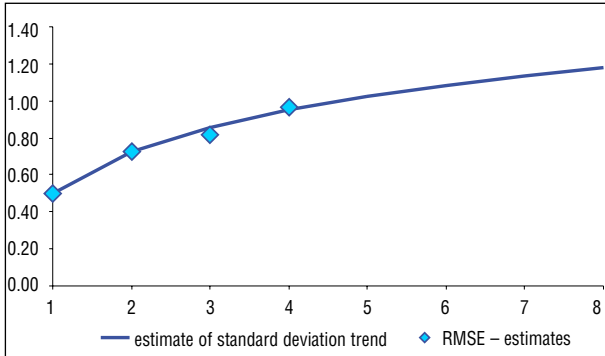
Table 1 Standard deviation estimate of the inflation forecast

1Q	2Q	3Q	4Q	5Q	6Q	7Q	8Q
0.500	0.729	0.819	0.963	1.024	1.083	1.133	1.177

Graph 2 Standard deviation estimate

estimate of standard deviation trend ◆ RMSE – estimates

Graph 3 Comparison of normal density function and distribution of errors in inflation forecasts (1Q)

distribution of forecast errors — density function
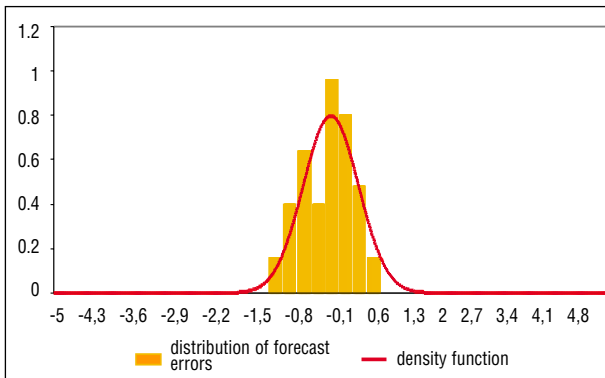
rithmic trend) was used for estimating standard deviations for quarters 5 – 8, i.e. the estimate of the forecast uncertainty for the second year ahead.

We note that in the first phase of the development of fan charts we focused on estimating symmetrical probability distributions. By making this assumption we have left out an estimate of the skewness and assumed that this probability distribution is symmetrical to the central view, i.e. the tendency of error to higher and lower inflation values is equally probable. The justification for this decision is clear from Graph 3, which depicts the distribution of error in a forecast for the first quarter ahead. It can be seen that forecast error is to a large degree distributed symmetrically around the most probable value, represented by the highest column in the histogram.

Conclusion

The National Bank of Slovakia will from 2005 present its inflation forecast in the form of fan charts four times per year as a part of its medium-term forecast of the economy's development. The presentation of the inflation forecast in this form will enable the central bank's view to be extended to include an estimate of the degree of uncertainty and risk around the point prediction. In case that no specific forecast risks are identified the distribution of uncertainty and risk shall be symmetrical, similar to the presented results. In the case of specific risks alternative scenarios may be prepared, whereby the degree of uncertainty may be shifted above or below the central view. In such a case the resultant interval would be asymmetrical around the point prediction. As we have said, in estimating forecast uncertainty, the econometric model will be an important instrument in the future. On the basis of simulations (deterministic and stochastic) it will be possible to estimate more precisely the degree of uncertainty and risk of an inflation forecast.

Normal Density Function

The random variable X has a normal distribution with the parameters μ and σ when its density has the form:

$$f(x) = (2\pi\sigma^2)^{-1/2} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right].$$

The basic property of the normal distribution is its symmetry to the central value. In this box we present the procedure used in expressing the density of the distribution's asymmetry. A simple and at the same time elegant way of introducing skewness is to define a two-piece normal distribution, based on two normal distributions with the same central value and different variances

$$f(x) = A \exp\left[-\frac{(x-\mu)^2}{2\sigma_1^2}\right] \text{, pre } \mu \quad x \qquad\qquad f(x) = A \exp\left[-\frac{(x-\mu)^2}{2\sigma_2^2}\right] \text{, pre } \mu > x,$$

where $A = \left(2\pi\left(\frac{\sigma_1 + \sigma_2}{2}\right)^2\right)^{-1/2}$. An alternative method of introducing asymmetry into the formula for the normal density function is the introduction of a new parameter – skewness. Density takes the following form:

$$f(x) = \frac{2}{(1-\gamma)^{-1/2} + (1+\gamma)^{-1/2}} (2\pi\sigma^2)^{-1/2} \exp\left[\frac{-1}{2\sigma^2}\left((x-\mu)^2 + \gamma\left(\frac{(x-\mu)}{|x-\mu|}\right)(x-\mu)^2\right)\right],$$

where σ represents variance error and γ skewness.

Literature used:

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